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METEOROLOGICAL OFFICE

Scientific Paper No. 21

Estimation of Rainfall using Radar—a critical review

by T. W. HARROLD, B.Sc., D.I.C.

LONDON: HER MAJESTY'S STATIONERY OFFICE

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NOTATION

A = cross-section area of radar antenna
A_e = effective antenna area, assumed equal to $\frac{2}{3}A$
b = length of short side of tetrahedral corner reflector
B,β = empirical constants defined by $Z = BR^{\beta}$
D = diameter of raindrop in cm
h = radar pulse length in μs (1 microsecond is equivalent to 300 metres in space)
k = one-way attenuation in dB/km
L = luminance of radar display, expressed in units of light energy $\times$ s^{-1} $\times$ area^{-1}
m = complex refractive index of the scattering particles
M = water-vapour content of air in g/m^3
N_D = number of drops of diameter D per m^3
P_t = power transmitted from aerial
P_r = power received at aerial
$Q(D,\lambda)$ = total absorption cross-section of drop of diameter D
R = rate of rainfall in mm/h
r = range
$\overline{V}_t$ = voltage received at aerial
$\overline{V}_r$ = $\overline{V}_t$ corrected for range
W = liquid water content of air in g/m^3
Z = reflectivity factor, defined by ΣD^6 mm^6/m^3
$|K|$ = $|(\epsilon-1)/(\epsilon+2)|$ where ϵ is the dielectric constant of the scattering particles
λ = wavelength
θ_1, ϕ_1 = beam widths in horizontal and vertical to half-power points
σ = back-scattering cross-section of a raindrop.

Note: Other symbols are as defined in the text.
Units are c.g.s. unless otherwise stated.
Power ratios are expressed in decibels (dB) where

$$\text{dB} = 10 \log_{10} \frac{P_1}{P_2}$$

Estimation of Rainfall using Radar—a critical review

by T. W. Harrold, B.Sc., D.I.C.

INTRODUCTION

Over the last decade several investigators have considered the possibilities of making quantitative rainfall measurements using radar. If sufficiently accurate, such measurements might be of great value in hydrology where one of the problems is to determine the amount of water falling into a catchment during a given time period. It seems possible that with radar such information might be immediately obtainable over a wide area so that predictions of river flow and flooding might be quickly made. A recent World Meteorological Organization Technical Note[1]* has also stated the importance of investigating the use of radar in measuring precipitation over the sea. This paper considers the practical possibilities of these two aims.

There are two basic methods of making quantitative radar rainfall measurements. One of these relates the intensity of the echo received from the rain to the rate of rainfall. If Plan Position Indicator (PPI) scanning is used and the signals can be integrated over an area and period of time then this method should provide a measurement of areal rainfall. The alternative method depends on the fact that at certain wavelengths the attenuation of a radar signal as it passes through rain can be related to the mean rate of rainfall along the path. If the attenuation could be measured along several fairly closely spaced paths then it might be possible to estimate the rainfall over an area. This paper discusses the various factors which limit the accuracies of both these measurements and describes how a radar might be developed so as to register automatically the rainfall amount over a chosen area. The notation used is given on p. iv.

Part I. Measurement of Rainfall based on Echo Intensity

Theory.

The average power received from precipitation which uniformly fills the beam at range r is (Probert-Jones[2])

$$\overline{P_r} = \frac{\pi P_t h A_e}{256 \ln 2} \frac{1}{r^2} \sum \sigma \qquad (1)$$

where the summation is over unit volume. The derivation of this equation assumes that the pulse volume is uniformly filled by incoherent scatterers. Attenuation of the signal between the radar and echoing volume is not included in the equation here. Other assumptions, which it is thought introduce negligible error for most radars, are that the aerial is of circular cross-section and that the polar diagram of the beam can be represented by a Gaussian function.

* The superscript figures refer to the bibliography on p. 51.

When the raindrops are small compared with the wavelength, that is $D \ll \lambda$, then the Rayleigh approximation

$$\sum \sigma = \frac{\pi^5}{\lambda^4}\left|\frac{\epsilon - 1}{\epsilon + 2}\right|^2 \sum D^6 \tag{2}$$

can be made. The radar equation (1) then becomes

$$\overline{P_r} = \frac{\pi^6}{256 \ln 2} \frac{P_t \, h A_e |\mathrm{K}|^2}{\lambda^4 r^2} 10^{-12} Z = \frac{CZ}{r^2} \tag{3}$$

where C is a constant combining radar and refractive index parameters. Empirical relations of the form $Z = BR^\beta$ can be found, so that

$$\overline{P_r} = \text{const} \frac{R^\beta}{r^2} \cdot \tag{4}$$

This equation forms a basis for measuring the rate of rainfall. The rainfall amount over a period of time can be calculated by integrating the rainfall rate. The radar can be used to measure rainfall effectively at a point although it is in practice sometimes difficult to determine where the rain in the radar beam reaches the ground. A more practical and useful method is to measure rainfall over an area.

There are several meteorological factors which will limit the accuracy of the measurements and these, together with the restrictions imposed by the assumptions in equation (1) will be discussed in Part IA. In Part IB, the instrumental difficulties will be considered and the progress which has been made by several workers towards a fully automatic rainfall computing system will be summarized.

PART IA—METEOROLOGICAL LIMITATIONS

1. *Z–R relationship at the ground*

A fundamental limitation to the accuracy of the rainfall estimate is the purely empirical nature of the relationship between Z and R. Since there is no unique drop-size distribution for a given rate of rainfall there is no unique Z–R relationship. Many relationships have been calculated from drop-size distributions measured at the ground. The simplest and most used method of obtaining the distribution is to expose a dyed filter paper to the rain. However the Illinois State Water Survey have made extensive use of a raindrop camera, which photographs a volume of the rain. In the calculations the fall speeds of the drops are required and these are obtainable in published tables. Because of the smallness of the volume sampled, typically of the order of a cubic metre, both methods of sampling may be inaccurate at larger diameters, to which Z is more sensitive.

The variation of the Z–R relationships is reduced if the data are grouped into different rain types. One of the simplest divisions is into continuous, showery and thundery rains. Very little improvement is obtained by subdividing further into synoptic types (Johnson[3]).

Z–R values have been calculated from drop-size distributions measured by Andrews. The data were obtained on the roof of the Imperial College of Science and Technology, London, using the filter paper technique. A few of the distributions have been published by Mason and Andrews,[4] together with a description of the experimental method. Since it is desired to measure the rainfall rate from the radar echo power return, Z is selected as

the independent variable, making R the dependent variable. The appropriate regression is that of log R on log Z and this is shown for the three rain types in Table I. The regression of log Z on log R is shown for comparison. It will be seen that there is a high correlation between log R and log Z.

TABLE I. *Z–R relationships from Andrews drop-size distributions*

Rain type	Z–R relationships obtained by regression of log R on log Z	log Z on log R	No. of points	Maximum R mm/h	Linear correlation coefficient of log R and log Z
Continuous	$Z = 204R^{1.52}$	$Z = 203R^{1.45}$	157	17	0·97
Showery	$Z = 280R^{1.46}$	$Z = 282R^{1.34}$	108	35	0·96
Thundery	$Z = 280R^{1.71}$	$Z = 485R^{1.40}$	66	120	0·90
All	$Z = 215R^{1.60}$	$Z = 266R^{1.47}$	331	120	0·96

Z–R relationships for different parts of temperate latitudes have been quoted by several authors but although a least squares method was used it is not generally stated whether a regression of log Z on log R or vice versa was calculated. Jones[5] thought that previous workers had calculated the former, which, as shown above, is less suitable for the type of measurement we are considering. Figure 1 compares Z–R relationships published by several authors using differing methods of obtaining the drop-size distributions. Bearing in mind the difficulty of obtaining representative drop-size samples and the uncertainty as to which regression was published by some authors, this figure shows that Andrews' data are in reasonable agreement with those obtained in various parts of the temperate latitudes. However, some variation between localities would be expected.

The scatter in the Z–R relation is of fundamental importance in assessing the possible accuracy of a radar measurement of rainfall. This scatter can be considered as made up of the variation of individual observations about the regression for any one storm and the variation of regressions between storms.

The standard error of an estimate of R in a particular storm, using the regression calculated for that storm, is less than 10 per cent (e.g. Johnson[3]). In an estimate of rainfall over a period of time there would be a tendency for successive errors in R to cancel out. Hence we can conclude that the scatter of Z–R points about the mean regression for any one storm is unlikely to produce a serious error in an estimate of rainfall.

Storm-to-storm variations can be compared by calculating the regression for each storm. Such regressions have been calculated for all storms measured by Andrews during which at least five filter paper exposures were made, this being sufficient to enable a log R/log Z line to be drawn by eye. They are shown in Table II. The low number of observations, sometimes made over less than half an hour, may overestimate the variation of the Z–R relationship between storms, since the line plotted may not be representative of the entire storm. The envelopes of these regressions for continuous rain, showers and thunderstorms, together with the mean regressions, are shown in Figure 2. Regressions of log R on log Z for individual storms have also been calculated by Jones[5] and the spread in these is shown for comparison in Figure 3. The drop-size distributions on which Figure 3 is based were obtained in Illinois with the raindrop camera, and generally include a larger number of points in each storm compared with the Andrews material.

In order to compare the relationship of individual storms with the mean relationship for the appropriate rain type it is necessary to make the comparison at a given Z. Unfortunately there are only sufficient British data to make a quantitative comparison for

TABLE II. *Z–R relationships in individual storms. London*

Date	Z	Number of observations	Range of R mm/h	k at 0·86cm*
(*a*) Continuous rain				
6.3.56	$210\ R^{1·3}$	5	0·07–0·63	$0·25\ R^{1·16}$
13.4.56	$160\ R^{1·6}$	5	0·07–2·82	$0·24\ R^{1·23}$
29.5.56	$130\ R^{1·7}$	13	0·21–3·77	$0·25\ R^{1·11}$
4.6.56	$180\ R^{1·75}$	10	0·30–1·13	$0·25\ R^{1·21}$
13.7.56	$500\ R^{1·6}$	5	0·11–1·22	$0·28\ R^{1·03}$
1.10.56	$170\ R^{1·6}$	19	0·47–17·1	$0.23\ R^{1·11}$
14.11.56	$320\ R^{1·5}$	5	0·93–4·28	$0·28\ R^{1·01}$
29.1.57	$230\ R^{1·6}$	5	0·25–1·37	$0·28\ R^{1·23}$
4.6.57	$250\ R^{1·65}$	5	0·07–2·30	$0·23\ R^{1·10}$
16.7.57	$130\ R^{1·6}$	19	0·18–6·26	$0·22\ R^{1·12}$
24.2.58	$150\ R^{1·4}$	16	0·30–5·88	$0·25\ R^{1·08}$
27.3.58	$290\ R^{1·7}$	6	0·41–1·60	$0·27\ R^{1·09}$
(*b*) Showers				
22.3.56	$335\ R^{1·6}$	10	0·08–3·43	$0·28\ R^{1·06}$
14.6.56	$660\ R^{1·5}$	6	0·10–1·30	$0·26\ R^{1·00}$
28.6.56	$250\ R^{1·4}$	7	0·33–3·92	$0·26\ R^{1·14}$
27.8.56	$220\ R^{1·75}$	12	1·01–6·58	$0·27\ R^{1·01}$
1.10.56	$180\ R^{1·6}$	6	1·00–35·3	$0·26\ R^{0·97}$
20.3.57	$80\ R^{1·9}$	5	1·81–8·95	$0·23\ R^{1·10}$
26.3.57	$270\ R^{1·2}$	7	0·05–16·9	$0·28\ R^{1·00}$
(*c*) Thunderstorms				
29.8.56	$95\ R^{1·75}$	6	15·0–59·3	$0·47\ R^{0·84}$
6.9.56	$610\ R^{1·5}$	6	0·07–5·22	$0·23\ R^{1·00}$
16.5.57	$425\ R^{1·5}$	17	0·82–74·7	$0·27\ R^{0·95}$
3.6.57	$425\ R^{1·55}$	10	0·37–41·0	$0·27\ R^{0·96}$
12.8.57	$200\ R^{1·5}$	10	0·05–7·91	$0·25\ R^{1·05}$

* See Part II.

occasions with continuous rain. Table III shows this comparison, together with similar results for Illinois showers and thunderstorms. Although the mean relationships differ, the mean deviation of R in the three categories is very similar.

TABLE III. *Deviation of R about mean R deduced from Z–R relationship*

Location	Rain type	Mean deviation of R expressed as percentage of mean R	Maximum deviation of R expressed as percentage of mean R
London	Continuous	26 per cent (at $Z = 200$)	78 per cent
Illinois	Showery	24 per cent (at $Z = 400$)	95 per cent
Illinois	Thundery	22 per cent (at $Z = 4000$)	100 per cent

It is concluded from the individual regressions and Table III that the error arising from drop-size variations in estimating, from known values of Z, the rainfall at a point during a storm using the Z–R relationship for the particular rain type and locality would be at worst (2 cases in 46 in Illinois, no cases in 24 in London) 100 per cent of the mean R. The mean error would be about 25 per cent.

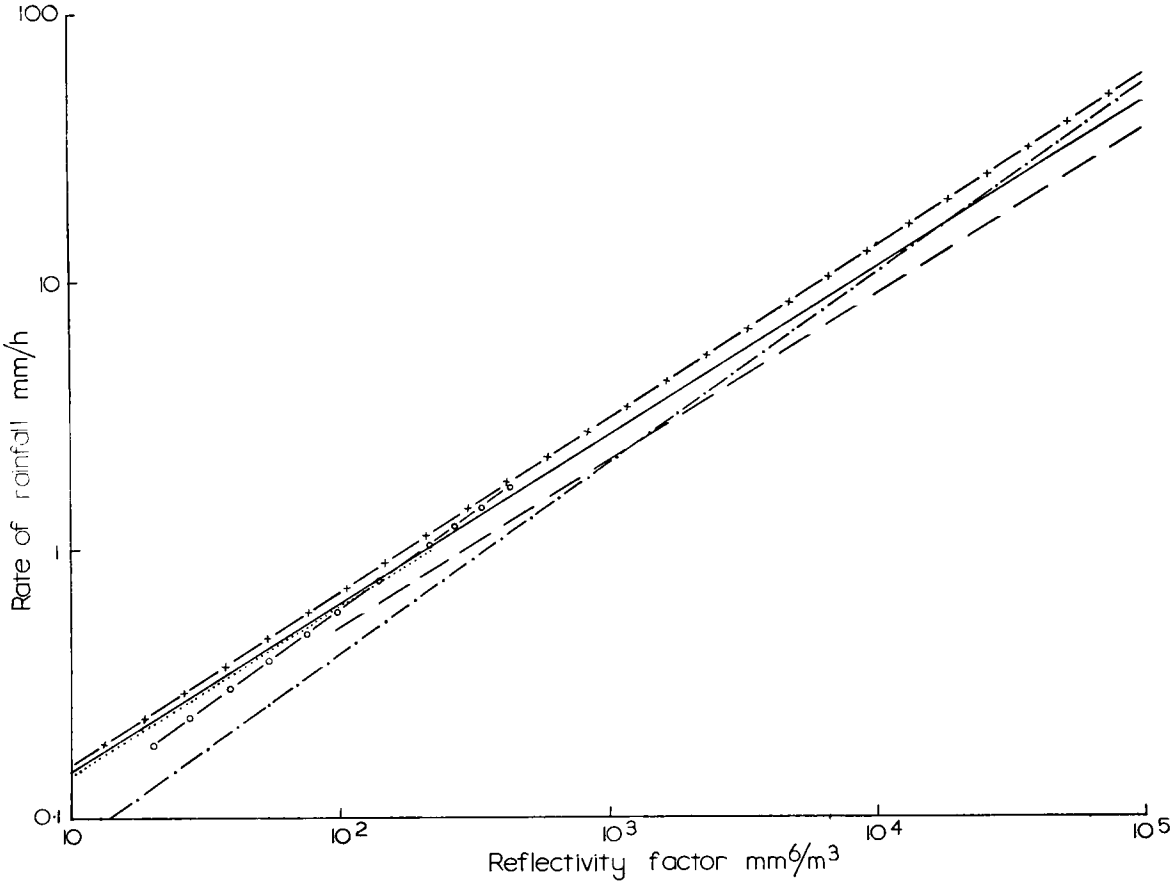


FIGURE 1. Examples of Z–R relationships in temperate latitudes.

Source		*Location*	Z
———	Andrews	London	$215\ R^{1\cdot60}$
———	Laws and Parsons[6]	Washington	$214\ R^{1\cdot58}$
·—·	Jones[5]	Illinois	$355\ R^{1\cdot41}$
— —	Hood[7]	Toronto	$295\ R^{1\cdot61}$
....	Marshall *et alii*	various	$220\ R^{1\cdot60}$
o—o	Muchnik[9]	Kiev	$207\ R^{1\cdot39}$
x—x	Boucher[10]	Massachusetts	$180\ R^{1\cdot55}$

Only Z–R data from temperate latitudes have been considered. Few observations have been published for warmer climates, but the Illinois State Water Survey are in the process of measuring drop-size distributions with raindrop cameras in several tropical and sub-tropical sites throughout the world.

In an estimate of rainfall over an area the spatial variations of the Z–R relationship are important. Very little has been published on this subject. It is possible that some of the extremes in point relationships may be smoothed out but there is no certainty of this.

2. *Rayleigh approximation and non-spherical drops*

The derivation of equation (4) involved the assumptions that the drops are small compared with the wavelength and are spherical. If these assumptions are not valid then either a relationship between $\Sigma\sigma$ and R must be calculated, or the Z–R relationship must be suitably corrected.

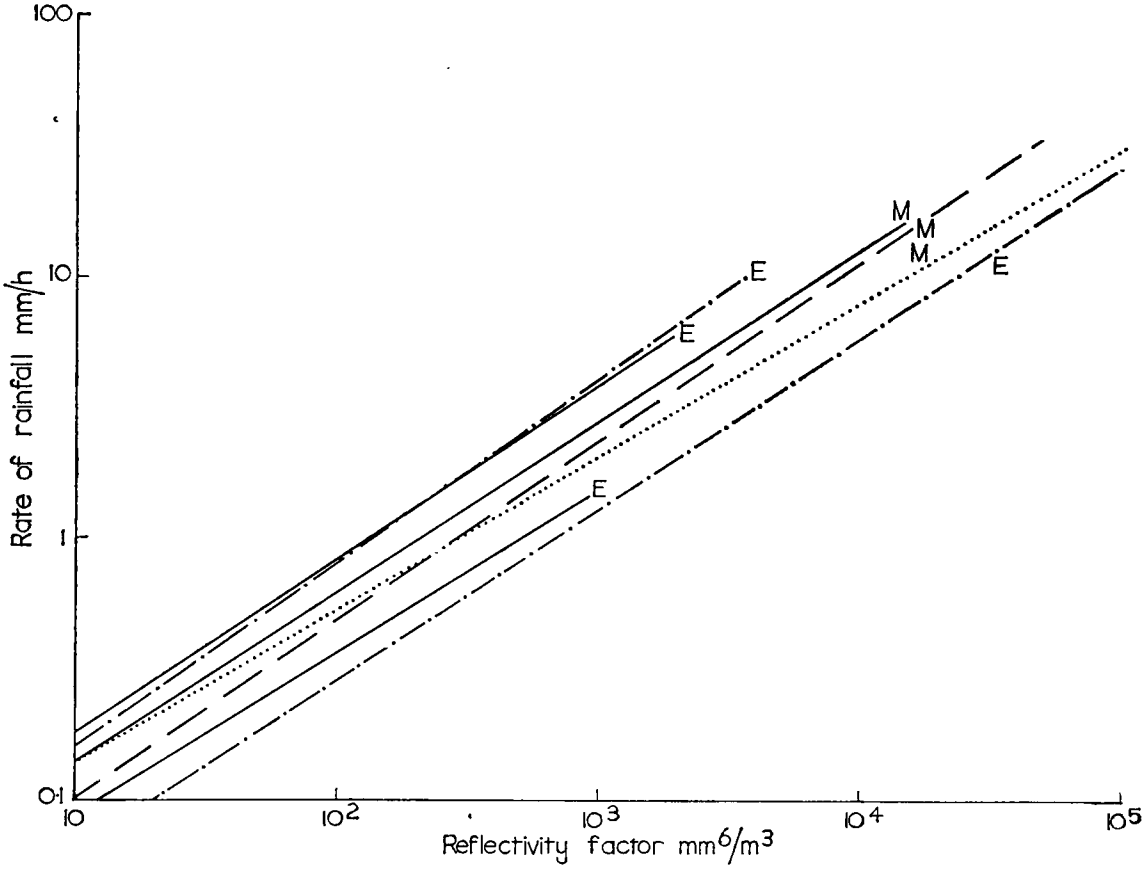


FIGURE 2. Mean Z–R relationships (M) and envelopes of relationships (E) from individual storms: London.

—continuous rain, — — showers, ... thunderstorms, —·—· showers and thunderstorms (limits coincide).

The error resulting from the use of the Rayleigh approximation, in place of the more exact Mie theory of scattering by spheres, has been calculated by Kodaira and Aoyagi.[11] Copies of their graphs of $Z_{\mathrm{MIE}}/Z_{\mathrm{RAY}}$ against R are shown in Figure 4. The calculations assumed a Marshall and Palmer[12] distribution of drop-size $N_D = N_0\, e^{-\Lambda D}$ where N_0 and Λ are constants. This distribution approximates to the drop-size distributions occurring in practice except that it overestimates the number of small drops. The other assumption made in the calculations was $Z = BR^{1\cdot6}$, which we have seen is a satisfactory approximation. At 10-cm wavelength the graphs show that the use of the Rayleigh approximation introduces negligible error at all rates of rainfall up to 100 mm/h. At 3·2 cm the error in Z_R increases at higher rates of rainfall, for example with $B = 200$ it rises to above 1 dB at more than 15 mm/h; that is, the use of the Rayleigh approximation will result in a small overestimation of heavy rainfall. It will be seen in Section 4 that this effect is generally negligible compared with the errors which arise because of the attenuation of the signal in rain.

When larger drops fall they become flattened. At any one time, therefore, a fraction of the total number of drops in any volume are non-spherical. Atlas *et alii*[13] applied Gans' theory of scattering by small ellipsoids to determine the effect of non-sphericity on the power received from rain. The results depend on the polarization of the radar beam, but generally the effect would not be significant in a radar rainfall measurement.

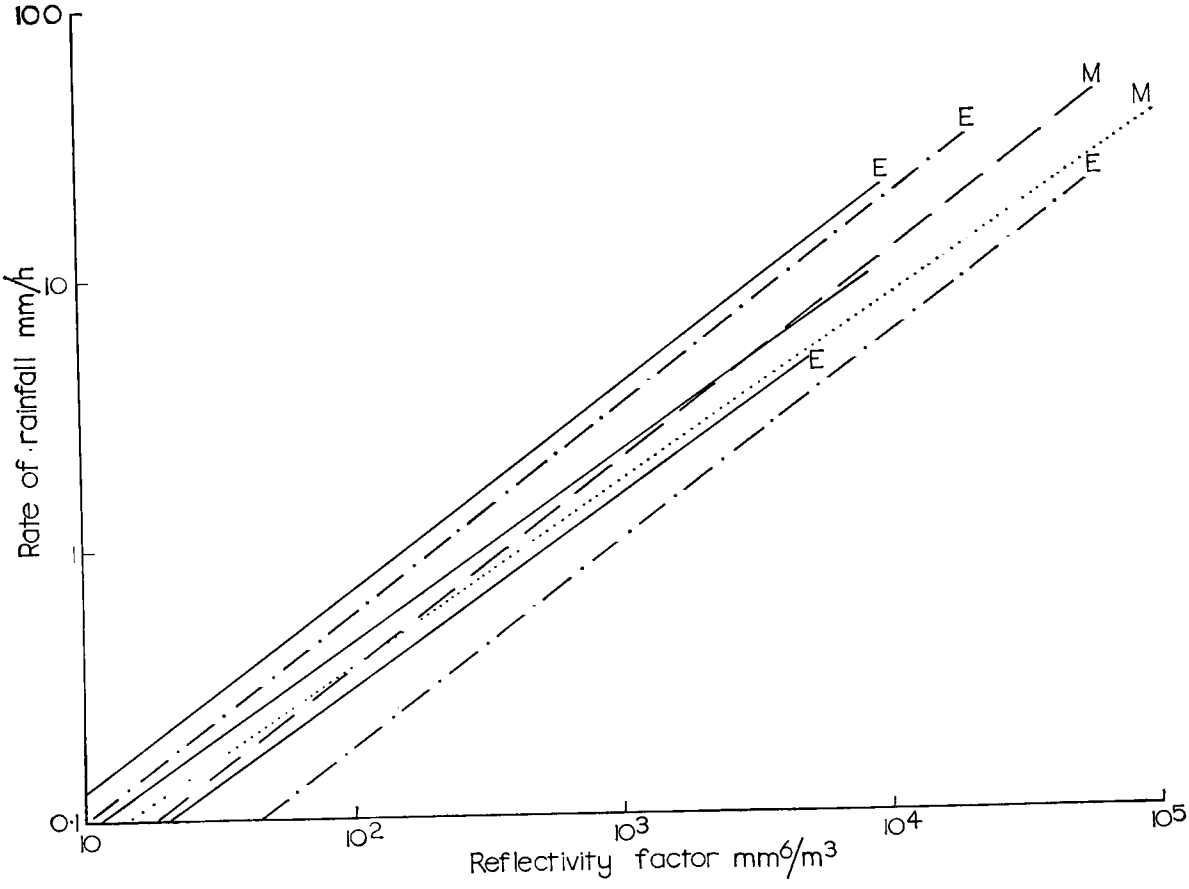


FIGURE 3. Mean Z–R relationships (M) and envelopes of relationships (E) from individual storms: Illinois.

— continuous rain, — — showers, ... thunderstorms, —·— showers and thunderstorms (limits coincide).

3. *Variation of Z with height in rain, and Z–R relationship aloft*

In order to measure rainfall at the ground using radar the reflectivity factor at the height of the radar sample must be related to the rate of rainfall at the ground. Since the drop-size distribution changes as the rain falls, Z and R are both a function of height. Mason and Ramanadham[14] and Hardy[15] have calculated the effects of coalescence of the drops, accretion of cloud droplets and evaporation. Hardy concludes that an initial exponential drop-size distribution $N_D = N_0\, e^{-\Lambda D}$ having a relatively large negative slope (i.e. Λ large) is considerably modified in falling owing to coalescence, leading to an increase in signal intensity whereas a distribution with a relatively small negative slope is only slightly modified.

Harper[16] averaged the radar signal intensity of 32 scans made on three days of widespread rain and his observations showed no significant increase in signal intensity in a

fall of 1 km. It is possible that the cloud water content was low, thus reducing the effect of accretion on these occasions. No increase of signal intensity due to coalescence was detected, suggesting that this increase does not exceed 1 dB/km. However, Caton,[17] using a vertically-pointing 3-cm Doppler radar in light to moderate steady rain found a mean

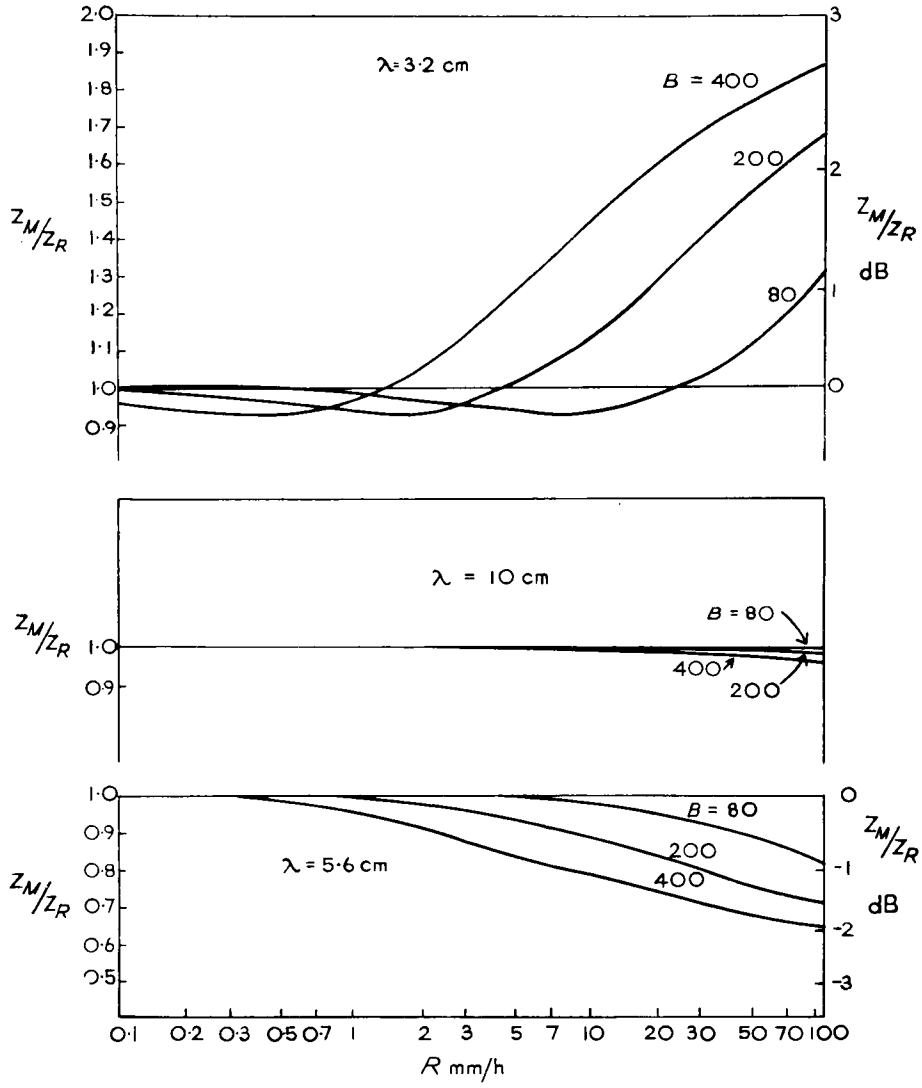


FIGURE 4. The ratio of the radar reflectivity Z_R to the reflectivity Z_M (after Kodaira and Aoyagi[11]).

increase of signal downwards of about 1 dB/km, this being largely independent of rate of rainfall. In a rainfall measurement such a change of Z with height could if necessary be allowed for by introducing a correction proportional to the height of the beam or, to a sufficient degree of accuracy, proportional to range. Thus for a beam at 1° elevation the correction would be +1 dB for each 45 km range. Since there is a scatter in the variation of Z with height this correction will be more accurate over a period of time than in an instantaneous measurement. Few experimental measurements are available in other than steady rain. However Donaldson[18] has shown that in the mean the reflectivity in thunderstorms

varies little below 13,000 feet, so that probably the same correction to Z can be applied for all rain.

Caton[17] obtained 83 drop-size distributions at each of several levels in 10 cases of steady rain. He calculated Z–R relationships at these levels and extrapolated to obtain a relationship at the ground. This relationship, $285R^{1.30}$, is compared in Figure 5 with the relationship obtained from filter papers. For a quantitative comparison, Andrews' results should

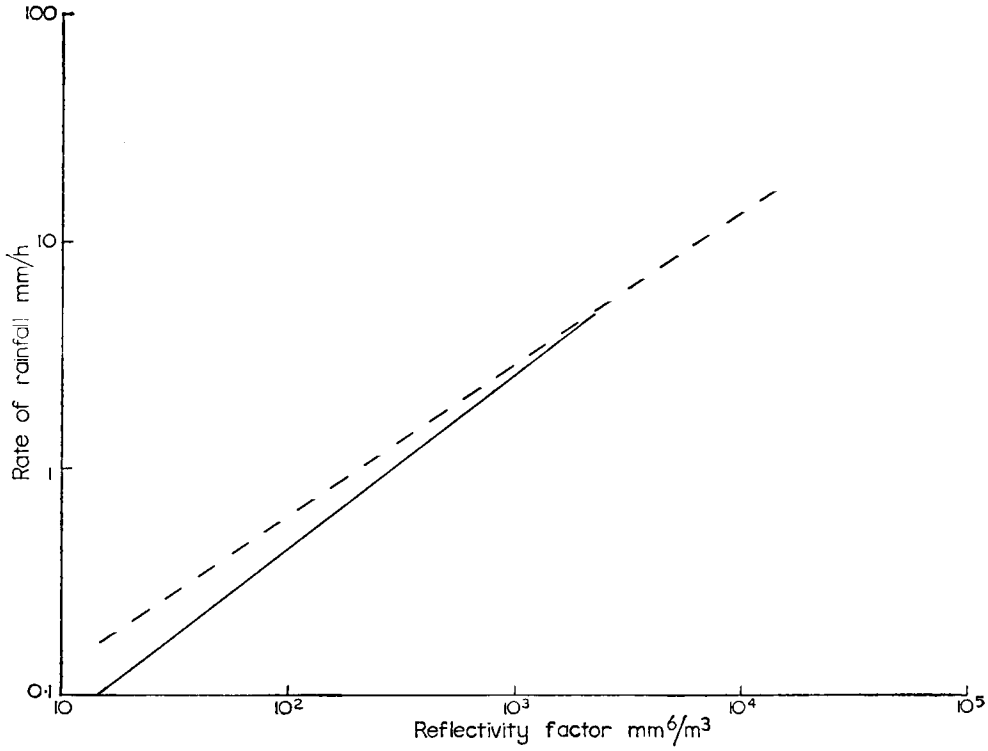


FIGURE 5. Comparison of filter-paper and Doppler Z–R relationships.
—— Doppler steady rains, — — — Andrews continuous rains.

be corrected for the approximations of Rayleigh scattering and spherical drops. For a vertically-pointing 3-cm radar these effects would increase the back-scatter from larger drops, which would increase the reflectivity slightly at higher rates of rainfall in Andrews' results. It is seen that above 1 mm/h, the two lines are in satisfactory agreement but towards low rates of rainfall they diverge. The reason for this difference at low rates of rainfall is not yet known. Fortunately the rainfall rates above 1 mm/h are the more important in the type of rainfall investigation that we are considering.

The Doppler measurements do not extend to above 5 mm/h and were made only in steady rain. Since only this limited sample is available, with no data in heavy continuous rain or other rain types, it seems preferable to use the Z–R relationships calculated from the filter papers throughout. This admits the possibility of error at low rates of rainfall in continuous rain but in a measurement of rainfall amount over an area and period of time this error will usually be small.

The scatter of points about the mean Doppler line was less than that from the filter paper observations. This may be because only steady rains were measured, but a contributing factor is probably the larger volume sampled, typically several thousand cubic metres

compared with about one cubic metre sampled by the filter papers. Also there were only small differences between the results on different days. This suggests that the spread of regressions in Figures 2 and 3 may be more than occurs aloft, so that Table III would then be an overestimate of the mean error in a rainfall measurement. Further evidence on this point is needed.

Summing up these three sections, Z–R relationships have been obtained from filter paper exposures at the ground. At wavelengths greater than 5 cm these relationships can be substituted directly into the radar equation, but below this wavelength small corrections due to errors in the Rayleigh approximation must be applied. The larger errors due to attenuation are discussed in the next section. Confirmatory evidence of the relationship for continuous rains has been obtained over the range 1 to 5 mm/h from the Doppler radar but there is some disagreement at lower rates of rainfall. It is more convenient in the discussions in this paper to consider one exponent for all three rain types, $Z \propto R^{1.6}$. This will be especially convenient in some of the circuitry developments considered in Part IB of this paper. The relationships derived from Andrews' drop-size distributions then simplify to the following:

$200R^{1.6}$ in continuous rain
$280R^{1.6}$ in showers
$350R^{1.6}$ in thunderstorms.

If it is not necessary for the exponent to be the same then the regressions of log R on log Z in Table I should be used. The mean error in an estimate of rainfall during a storm of known values of Z can be expected to be about 25 per cent.

4. *Attenuation of the signal*

A radar signal is attenuated by atmospheric gases, water vapour, cloud and precipitation. Attenuation by the first three of these is small compared with the attenuation in precipitation (see Part II, p. 41) and will not be discussed here.

The attenuation in rain can be related empirically to the rate of rainfall. Table IV (from Gunn and East[19]) shows these relationships for different wavelengths. These expressions have been partially verified by experiment (e.g. Robinson,[20] Robertson and King[21]) and will be used in considering the effect of attenuation on quantitative measurements of precipitation echo.

TABLE IV. *Relationship between attenuation and rate of rainfall*

λ cm	10	5·7	3·2	1·8	0·9
One-way attenuation (k) in dB/km	$0{\cdot}0003R$	$0{\cdot}0022R^{1.17}$	$0{\cdot}0074R^{1.31}$	$0{\cdot}045R^{1.14}$	$0{\cdot}22R$

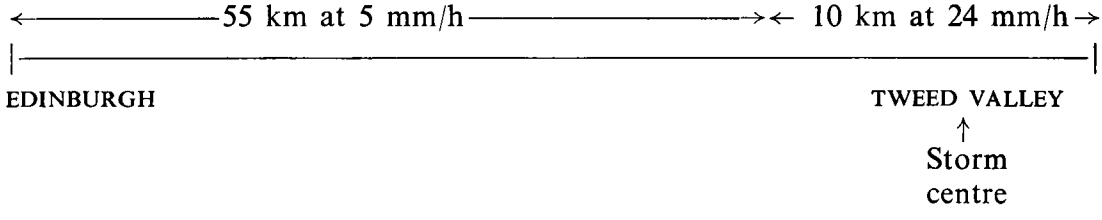
At 10 cm the attenuation is small except at very high rates of rainfall. Thus even when R = 100 mm/h, k = 0·03 dB/km and the two-way attenuation is less than 2 dB up to a range of over 30 km. Even in the tropics R = 100 mm/h over 30 km would be most uncommon. So at this wavelength attenuation will not be important.

At 1·8 cm the attenuation becomes appreciable in moderate rates of rainfall over short ranges. A signal is reduced to half its value (by 3 dB) in 33 km of rain at 1 mm/h or in less than 2 km of rain at 10 mm/h. This and shorter wavelengths are therefore not suitable for quantitative measurements of precipitation based on measuring the intensity of the echo.

Radiation at 3·2 cm has been used by many workers in rainfall investigations. Many (e.g. Hood,[7] Hitschfeld and Bordan,[22] Kodaira[23] and Hamilton and Marshall[24]) have advised against its use in quantitative measurements of precipitation intensity. For instance Hamilton and Marshall state "At 3·2 cm the quantitative measurements of rainfall that it had been hoped to gain were seriously impaired by the attenuation of the 3·2-cm radiation by rainfall". They carried out a statistical survey of the attenuation which would occur during summer showers in the Montreal region using 3·2-cm (and 5·7-cm) wavelengths. One of the conclusions was that the season's rainfall from the heavier storms (peak intensity above 40 mm/h) was 473 mm, but at a range of 48 km a 3·2-cm radar would only have measured 144 mm. (A 5·7-cm radar would have measured 349 mm). These figures are not of course directly applicable to the British Isles. In the Montreal data 50 mm/h was exceeded in 0·3 per cent of the duration of the summer rainfall. At a typical station in England and Wales this rate is exceeded for about 5 minutes a year, or 0·02 per cent of the annual duration of rain (Ryde,[25] Figure 8, and Climatological Atlas of the British Isles*).

Although the attenuation would not be so serious in this country, it would still restrict the ranges over which measurements could be made. Typical moderate frontal rain would have an intensity of about 2 mm/h, and even if we can accept an attenuation of $1\frac{1}{2}$ dB, rainfall measurements could be made only to a range of 40 km.

In heavy widespread rain however, the attenuation would lead to a completely false impression of the distribution of rain and in the extreme could even result in the apparent peak being in the wrong watershed (e.g. Mueller *et alii*[26]). Precise data for an intense widespread storm in this country are not available but some approximate figures have been determined for the storm which produced serious flooding in the Tweed Valley on 12 August 1948 (*British Rainfall* 1948, p. 43). From the information published in this reference the distribution shown in the following diagram seems a reasonable approximation to the instantaneous rainfall between Edinburgh, the nearest city, and the centre of the storm. This is probably an extreme picture for Great Britain.



The 3·2-cm radar signal would be attenuated by 12 dB at the storm centre. This would result in an apparent rate of rainfall of $4\frac{1}{2}$ mm/h compared with the actual rate of 24 mm/h. It is clear therefore that a 3·2-cm radar in the nearest city would have been very seriously misleading in its estimate of the severity of this storm. At 10 cm the corresponding attenuation would have been 0·2 dB, at 5 cm 2·5 dB.

* London, Meteorological Office. Climatological Atlas of the British Isles. London, HMSO, 1952.

The attenuation can also be appreciable in localized rain. Figure 6 shows a typical heavy shower as observed over the Cardington rain-gauge network. The shower has a diameter of about 4 km and a central intensity of about 100 mm/h. The centre of this shower would be attenuated by 6 dB if observed with a 3·2-cm radar from A, and by 8½ dB from B. Neglecting the effect of the finite beam width discussed in Section 11, this attenuation would reduce the apparent rate of rainfall at the core to 44 mm/h if viewed from A, and to 30 mm/h from B. Also from A, the 30 mm/h at C would appear as 5 mm/h. Thus even in storms of small spatial extent the attenuation can produce an underestimate of the intensity of the rain and a distortion of the shape of the isohyets.

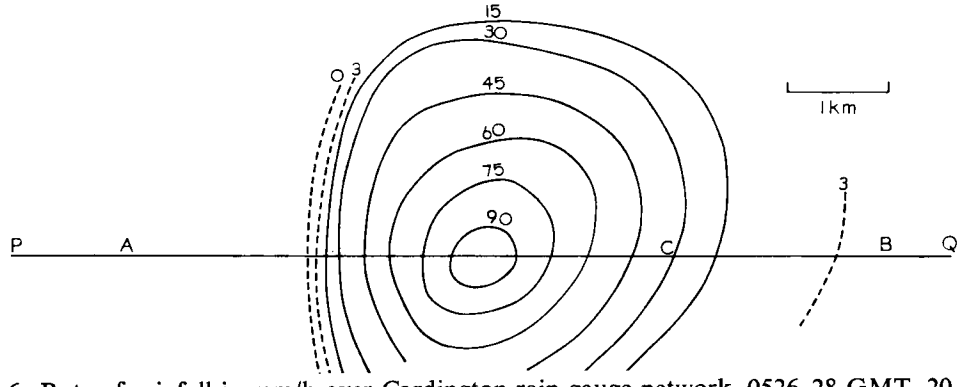


FIGURE 6. Rate of rainfall in mm/h over Cardington rain-gauge network, 0526–28 GMT, 20 July 1958

In the practical use of radar as an indicator of the rate of rainfall and, by time-integration, the amount of rain, the heavier falls would be of more interest than the 'normal'. It is in these cases that a 3·2-cm radar fails to give a true picture of either the intensity or area of the rain. Also radar 'shadows' are cast by precipitating cells which prevent detection of more distant storms. Even in moderate frontal rains the range out to which quantitative measurements can be made is reduced.

Even at 5·7 cm attenuation produces some distortion of intense rain patterns, but the effect is much less than at 3·2 cm. In view of the additional requirements of a narrow beam width discussed in Section 6, 5·7 cm would be a suitable wavelength for rainfall measurements in Great Britain, but in countries where heavier rains occur 10 cm is essential.

Methods of correcting for the attenuation at 3·2 cm have been attempted, the most elaborate of which are automatic electronic correcting devices developed by Kodaira,[23] Mueller *et alii*[26] and Wein.[27] Essentially these consist of a feedback loop following a logarithmic amplifier (see Part IB) in the intermediate frequency circuitry. As the radar pulse moves outward from the antenna an accumulating voltage, of magnitude dependent on the intervening precipitation, is added to the signal received. None of the authors appeared to have much confidence in this device. Kodaira for instance has not reported its further development since 1955 and the more recent Japanese weather radars are at 5·7-cm wavelength (Ohta *et alii*[28]) and these do not include a circuit for the correction of attenuation. Wein states that it is improbable that it will ever be possible to make the correction fully automatic—an operator's judgement is needed to set the attenuation calibration control, and he thinks that this would have to be done every half hour because of the fluctuations in the receiver and transmitter (see Section 13). This is because one of the greatest difficulties in any device for correcting the attenuation is that a very accurate knowledge of the radar sensitivity is required (Hitschfeld and Bordan[22]). As an illustration Wein

quotes an example in which a radar calibration error of only 0·8 dB produced an infinite value of the corrected reflectivity. It therefore appears to be more practicable to use a 5·7-cm or 10-cm radar rather than to attempt to correct for the attenuation at 3·2-cm.

5. *Variation of Z at and above the melting layer in frontal rain*

In frontal rains the radar beam, which we will assume to be at a small angle above the horizontal, will intersect the melting layer and pass into snow at larger ranges. A radar rainfall estimate then entails relating the combined reflectivity in snow, the melting layer and rain to the rate of rainfall at the ground. Figure 7 shows the height of a radar beam as a function of range for various elevation angles.

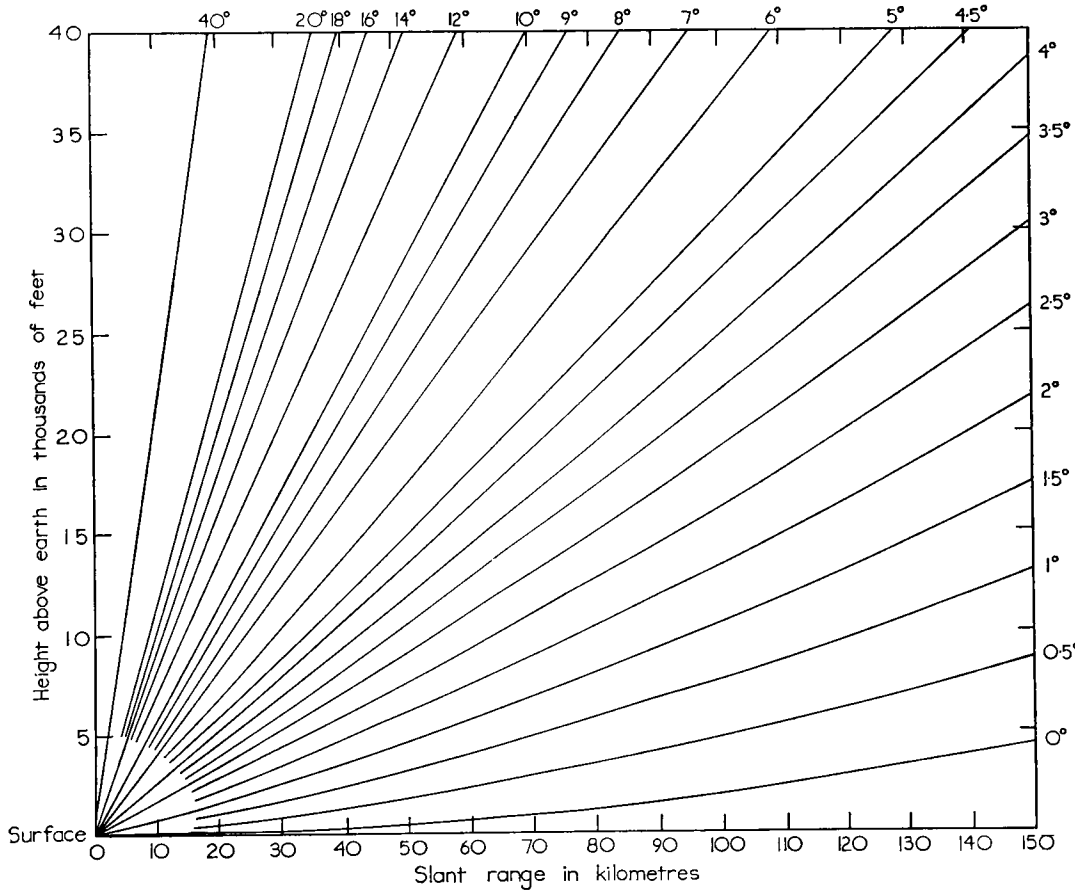


FIGURE 7. Approximate radar coverage diagram assuming standard refraction.

The apparent rainfall rate in beams passing through a model melting layer and snow region has been calculated. This simplified model of the variation of Z with height is shown in the inset to Figure 8. The main features are:

(i) constant Z below the melting layer;

(ii) melting layer 1000 feet deep with a peak reflectivity six times that in rain. In the author's experience this factor is very rarely exceeded in southern England;

(iii) decrease of reflectivity above the melting layer to zero 9000 feet above. The depth of the region of snow may at times exceed 9000 feet but the effect on the calculations will be small.

The profile is assumed constant regardless of the height of the melting layer. This model is a reasonable approximation to the well-marked bright bands that occur in steady rains.

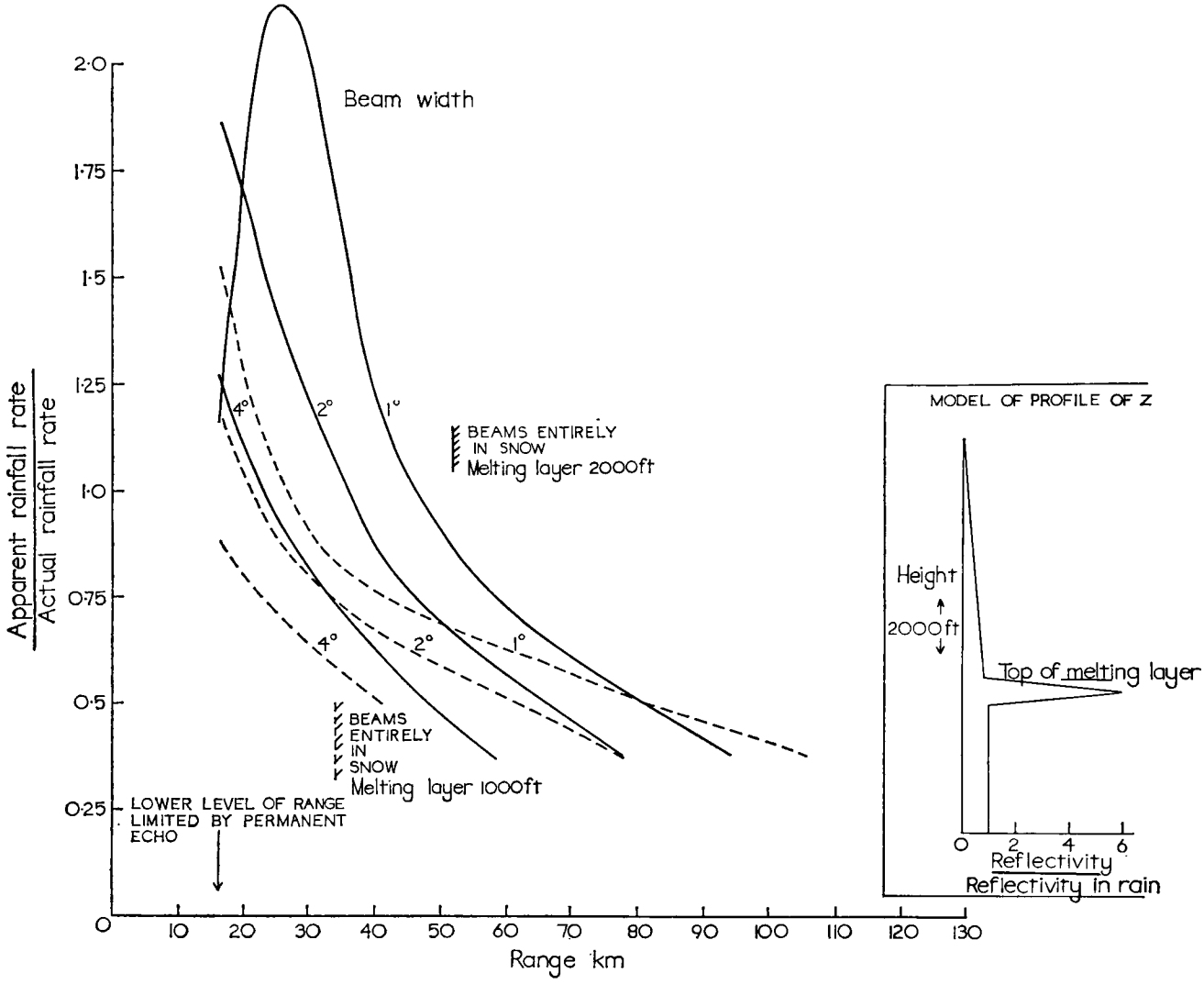


FIGURE 8. Apparent rate of rainfall in beams intersecting the melting layer. Inset: Model of profile of Z.
—— Top of melting layer 2000 ft, - - - - Top of melting layer 1000 ft.

Figures 8 and 9 show the apparent rate of rainfall R_a, compared with the rate of rainfall at the ground R_G, assuming $Z \propto R^{1.6}$. Results are shown for beams 4°, 2° and 1° wide in the vertical, all with the lower edge of the beam at $\frac{1}{2}$° elevation, and with the top of the melting layer at 1, 2 (Figure 8), 4, 6 and 10 thousand feet (Figure 9). The curves are not drawn at ranges less than 16 km since, as will be suggested in Section 9, measurements will probably not be practicable at these ranges due to permanent echoes. When below the melting layer the beam is uniformly filled with rain and $R_a = R_G$. As the range increases the top of the beam passes into the melting layer and R_a increases. With a further increase in range the top of the beam then passes into the snow region. The lower reflectivity in snow becomes

dominant until eventually beyond a certain range the entire beam lies above the precipitation. The magnitude of the increase of apparent reflectivity caused by the melting layer depends on the proportion of the beam filled by it. Thus R_a is greater in a smaller beam and for a lower bright band. The narrower the beam, the greater the range over which $R_a = R_G$.

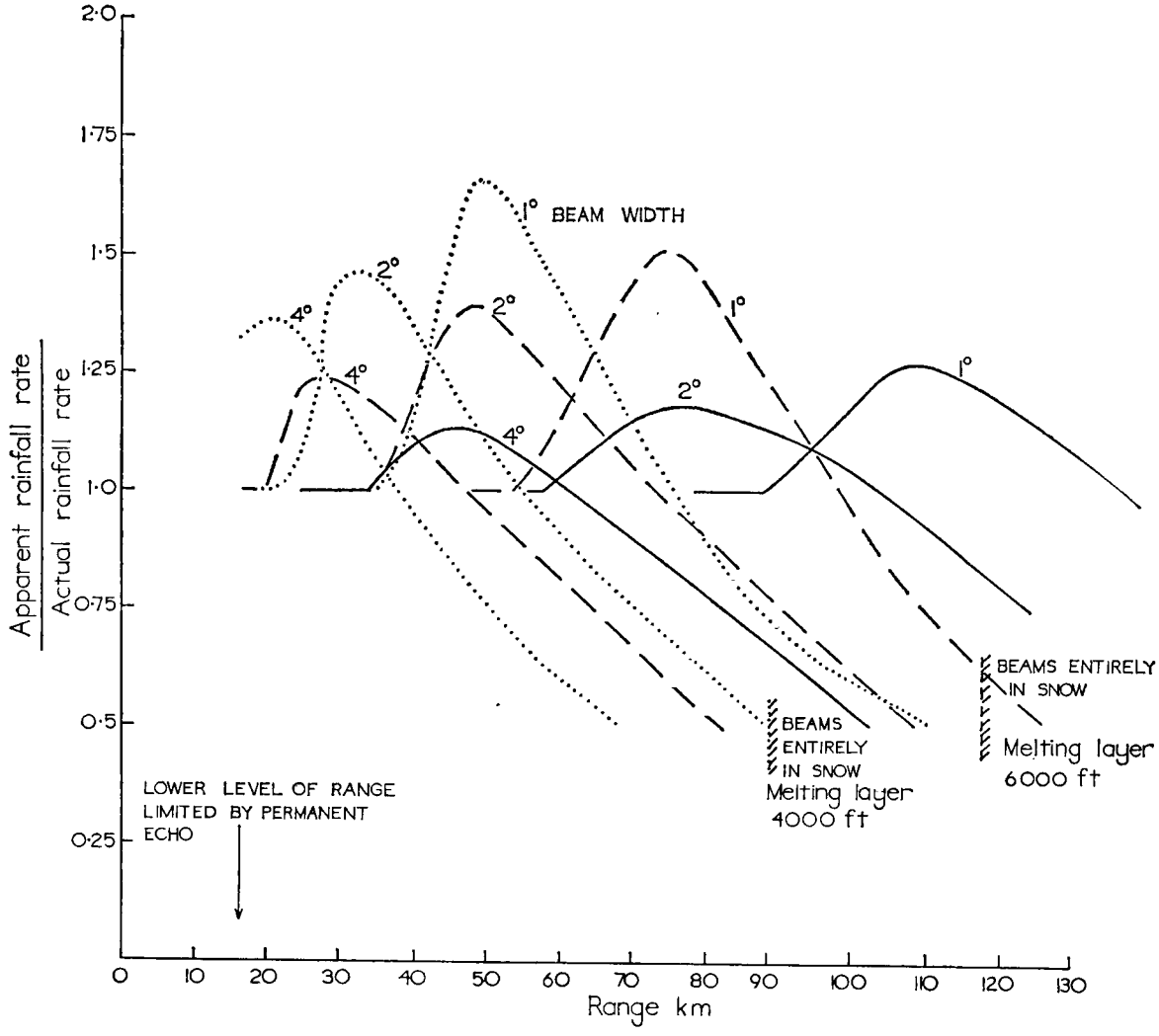


FIGURE 9. Apparent rate of rainfall in beams intersecting the melting layer.
— Top of melting layer 10,000 ft, – – – Top of melting layer 6000 ft, ... Top of melting layer 4000 ft.

Measurements of rainfall are best made when $R_a = R_G$, the initial horizontal portions of the graphs. Over these ranges the errors in a radar measurement of rainfall are largely those discussed in the first four sections. We shall assume that the measurement will still be a useful one over the range where $0{\cdot}5 < R_a/R_G < 2$. Thus when a 2° beam intersects a melting level at 6000 feet (Figure 9), the usable range is increased from 33 km to 110 km. In general, the horizontal range over which $0{\cdot}5 < R_a/R_G < 2$ is more than twice that over which $R_a = R_G$; in other words useful measurements can be made over twice the horizontal portions of Figure 9. Also the melting layer itself does not usually determine the upper limit to the range, which is affected more by the reflectivity in the snow.

The model assumed in the previous paragraphs provides a general description of the effect of the variations of Z with height in steady rain, but it must be emphasized that variations from this can be considerable. The model suggests that a correction, a function of range and melting level, might be applied to R_a when the beam is not entirely in the rain and this possibility will now be briefly explored. In considering the day-to-day variations of R_a, 103 actual profiles of Z, measured on 10 days, were used. The apparent rate of rainfall in a 2° beam, lower half-power point $+\frac{1}{2}°$, at a fixed range of 65 km has been calculated for these profiles. The mean value of R_a/R_G for each of the 10 days is shown in Figure 10. It can be seen that R_a/R_G is not well related to the height of the melting level, so that no simple statistical correction could be applied. In fact at this range when the melting level is above 5000 feet the variation of Z with height is best ignored in a rainfall measurement over a period of time and $R_a = R_G$ assumed. This helps to justify the assumption that useful measurements can be made beyond the range of $R_a/R_G = 1$ in Figures 8 and 9. Over the ranges when $0{\cdot}5 < R_a/R_G < 2$ the actual error in a rainfall amount may be less than the 50 per cent implied. Figure 10 suggests a mean error of about 25 per cent.

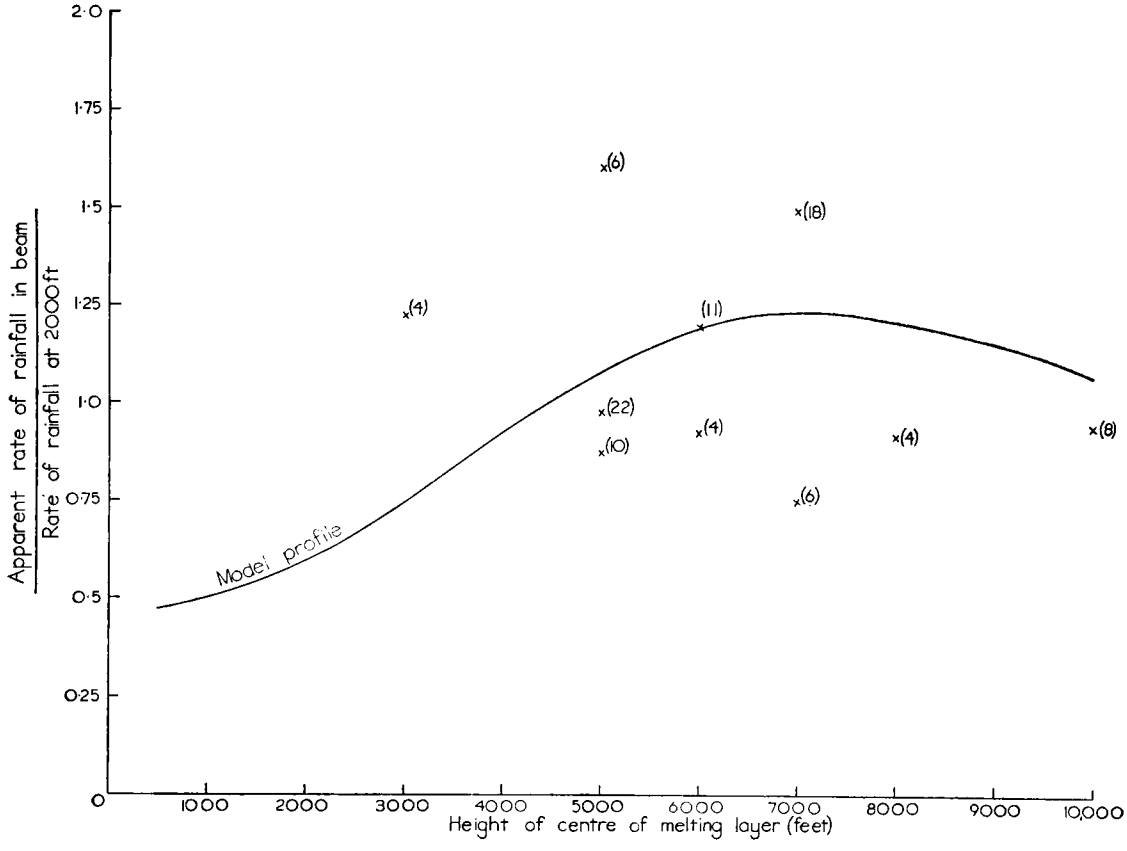


FIGURE 10. Mean apparent rate of rainfall in a 2° beam at 65 km on 10 occasions. The bracketed figures are the number of observations in each mean value.

6. *Vertical beam width*

Figures 8 and 9 show that estimates of rainfall can be made over greater ranges with a narrow beam in the vertical, and so as narrow a beam as possible is desirable. Measure-

ments will be seriously restricted with a beam width of much more than 2°. However, the narrower the beam the larger the aerial assembly and the greater the cost, so a compromise between beam width and cost may be necessary. Table V shows the approximate aerial diameter needed to produce a 1° and 2° pencil beam. It is assumed in this Table that

$$A_e = \frac{\pi\lambda^2}{4\theta_1\phi_1} = \frac{2}{3}A \quad \text{(after Probert-Jones[2])}.$$

The final choice of aerial size will also depend on the wavelength used. The choice of wavelength has been discussed in the section on attenuation (Section 4). We shall assume that our equipment can be provided with an aerial 3½ metres in diameter. This would provide a 2° beam at 10 cm and 1° at 5 cm. Examples of beam widths employed in operational weather radars are 1·8° for the American 10-cm WSR-57 and 1·5° for the 5·66-cm Japanese equipments.

TABLE V. *Diameter of aerial needed to produce stated beam widths*

λ cm	3	5	8	10
		diameter in metres		
$\theta_1 = \phi_1 = 1°$	2·12	3·51	5·62	7·02
$\theta_1 = \phi_1 = 2°$	1·06	1·76	2·81	3·51

7. *Variation of Z with height in convective rain*

In active convective conditions the vertical air currents will produce substantial local variations in drop-size distributions. However, Donaldson[18] found little variation in the mean reflectivity of thunderstorms below 13,000 feet. It seems probable therefore that over an area and period of time vertical air motion may not seriously affect a radar estimate of rainfall, but there will be localized errors. It is not possible to assess the magnitude of these because there is as yet no reliable information of the drop-size distribution aloft within strongly convective clouds.

In updraughts liquid water drops are carried above the 0°C level and can at times remain as supercooled water drops to temperatures at least as low as −20°C. A bright band is not detectable in these circumstances. To a first approximation we may consider that the horizontal portions of the lines in Figure 9 will be extended, though this may be modified locally owing to drop-size variations. In decaying convective cells, where updraughts are weak or non-existent, conditions are more similar to those in steady rain and a bright band is often present.

In general then Figures 8 and 9 show the error probable in measuring rainfall in frontal rain, and also approximate to conditions in decaying convective cells. The figures do not apply in active convective cells, when measurements could probably be made at larger ranges than the figures suggest, although the horizontal beam width (Section 11) may then become important.

8. *Limitations imposed by the melting level in practical measurements*

No statistics of the variation of height of the melting level with season have been published for Great Britain but relevant data have been extracted from published upper air records. Table VI shows the proportion of ascents from 1951–55 when the temperature was

below 0°C at various levels in south-east England and, for comparison, at Malta in a winter month (M.O. 628 parts 2 and 6[29]). These values are for all ascents and consequently are not an exact description of conditions in precipitation alone. For instance in south-east England it seems probable that in winter the mean 0°C level is lowered by cold and often dry easterly weather whereas most of the rain falls in the milder westerly type. Therefore the Table gives only a general indication of the conditions to be expected.

TABLE VI. *Percentage of occasions with temperature below 0°C*

	South-east England			Malta
Height (feet)	February	July	Year	January
15,000	100	100	100	100
10,000	100	40	78	94
7000	—	—	50	—
5000	80	1	33	5
2500	50	0	—	<1
0	20	0	5	0

When the melting level is at or below 2000 feet (Figure 8) which will sometimes be the case in winter in this country, the beam intersects the melting level at a range of less than 16 km. The accuracy of the rainfall measurement is therefore limited at all ranges, but approximate measurements could be made to about 60 km with a 2° beam, and perhaps 80 km with a 1° beam. (However, see Section 12 on minimum detectable signals.) With the top of the melting level at 4000 feet, a typical value for early spring and late autumn, a 1° beam can provide data to 110 km, a 2° beam to 100 km. In summer, typified by the full curve of Figure 8, a 1° beam intersects the melting level at nearly 90 km, a 2° beam at 55 km, and even beyond these ranges the error would be small. It must be emphasized then that in warmer climates and in summer in this country the effect of the beam passing above the rain is small and measurements would probably be practicable to at least 100 km.

9. *Site selection*

So far in this paper only that part of the beam bounded by the half-power points has been considered. A complete beam pattern includes side lobes as well as the main lobe, the two-way intensity level of the strongest side lobe being generally 20 to 30 dB below that of the main lobe. At low angles of elevation there will be returns from permanent echoes in either the main or, more usually, side lobes.

Site selection will almost certainly be necessary. The 'ideal' site is one in the centre of a shallow saucer-shaped depression with horizontal obstructions not exceeding $\frac{1}{2}°$ in elevation at a range of 10 to 15 km, so that the base of the beam can be held at a low elevation angle. Within this horizon range permanent echoes would be severe and rainfall measurements would not be possible. If an ideal site is not available it may be possible to obtain a suitable horizon in the sector of main interest.

A $\frac{1}{2}°$ horizon level would have the further advantage that only severe cases of anomalous propagation, such as occur in widespread and deep fogs, would affect the measurements. Fortunately anomalous propagation mostly occurs in meteorological situations in which rain is improbable, and even slight experience would enable one to distinguish the cases.

Permanent echoes could be greatly reduced by use of the constant altitude PPI presentation (CAPPI) developed by McGill University, Canada (East[30]). With this, echoes from approximately the same height interval are displayed on a screen. At short ranges a height clear of permanent echo could be used, but at larger ranges there is no advantage in CAPPI compared with using the beam at the minimum elevation so that it is as close to the ground as possible. It is the author's opinion that unless a radar is already equipped with CAPPI, the use of this additional equipment is not justified. However, when permanent echoes are more widespread than this a useful increase in the area of measurement can be obtained using CAPPI.

10. *Effect of side-lobe energy on precipitation measurement*

As an example consider the two-way beam pattern of a main and first side lobe shown in Figure 11, and approximate this by the dotted line. The complete pattern is assumed to be that obtained by rotating the diagram about the y axis. It can be simply shown that the area of the side lobe is 5 times that of the main lobe (A_m). For a given side lobe of, say, -20 dB, a point target in the main beam returns 100 times the power from the same target in the side lobe.

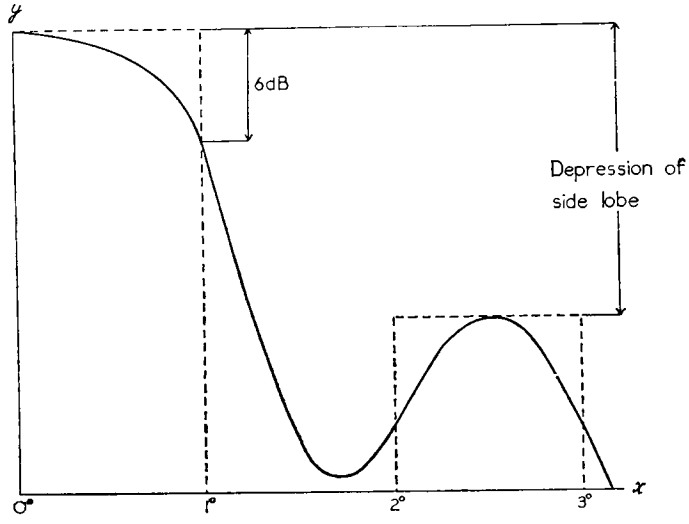


FIGURE 11. Diagram of Beam Pattern assumed in text.

Suppose uniform precipitation fills both the main and side lobes. Then with a -20 dB side lobe the power returned from such a lobe is 5/100 of that returned in the main lobe, or approximately 5 per cent of the total received power. Table VII shows the contribution from the side lobe in this and two other cases, to the total power received in uniform rain. It shows that the effects of the side lobe can be neglected, a conclusion also reached more rigorously by Probert-Jones.[2]

TABLE VII. *Contribution to total power from side lobes in uniform precipitation*

Depression of side lobe	15 dB	20 dB	30 dB
Contribution to total power	15 per cent	5 per cent	0·5 per cent

However, when the precipitation is not uniform this conclusion may not apply. As an example suppose a convection cell fills just 10 per cent of the side lobe and none of the main lobe, so that all of the received power is from the side lobe. In applying the radar equation we assume all of the power to be from an apparent rate of rainfall in the main lobe. Thus for a -20 dB side lobe, 10 per cent of which is filled with rain of scattering cross-section $\Sigma\sigma$ per unit area, the power in the side lobe is $\propto (10/100) \times 5 A_m \times \Sigma\sigma$. To obtain the same power in the main lobe we require a scattering cross-section of $f\sigma$ where

$$100 A_m f \Sigma\sigma = \frac{1}{2} A_m \Sigma\sigma \text{ or } f = \frac{1}{200}.$$

In other words the apparent cross-section in the main lobe is 1/200 of that actually in 10 per cent of the side lobe. Assuming $\sigma \propto R^{1.6}$ the apparent rate of rainfall is 0·035 of the actual rate. This and similar results are summarized in Table VIII. In an area rainfall measurement this effect may introduce an appreciable error when the precipitating cell is actually outside the area of measurement. There may appear to be rain within the area. In a radar rainfall measurement side lobes 20 dB or more below the main lobe are preferable, but useful results should be obtainable in all but the most extreme cases of non-uniform precipitation with side lobes 15 dB down (provided permanent echoes are not excessive).

TABLE VIII. *Example of the apparent rainfall in an area due to side-lobe detection of a cell outside that area.*

Depression of side lobe	15 dB	20 dB	30 dB
Apparent rainfall in main lobe / Actual rainfall in 10 per cent of side lobe	0·08	0·035	0·008
Rainfall rate in 10 per cent of side lobe giving apparent rate of 1 mm/h in main lobe	13 mm/h	28 mm/h	116 mm/h

11. *Effect of horizontal beam width on rainfall measurement*

In widespread uniform rain the horizontal width of the beam introduces no error in a rainfall measurement. However, this is not so if the rain is not uniform. Figure 6 shows the rate of rainfall in a typical heavy shower, described in Section 4. The apparent rate of rainfall in beams of different widths scanning through this shower from P to Q has been calculated. The results are shown in Figure 12. The full line represents the actual profile of the shower, the dotted and dashed lines show the rate of rainfall as measured by a radar with beam widths 2·45 km and 4·9 km at the shower. This figure shows that the rate of rainfall at the centre of the shower is underestimated, whereas that at the edge is overestimated. The mean rates of rainfall between P and Q are shown in Table IX. This shows that a 4·9-km beam scanning through the shower which is 5·5 km across only produces a 17 per cent error in the mean rate of rainfall between P and Q. If the shower is on the edge of the area of interest the error will be more but in a time average such errors would tend to cancel out. These calculations show that although a narrow beam in the horizontal is desirable it is not so important as a narrow vertical beam. Measurements can still be made to about 65 km in showery weather, and further in widespread rain with a beam as much as 4° wide, if an additional error of about 20 per cent can be tolerated.

12. *Minimum detectable rate of rainfall*

The strength of the minimum detectable signal is determined primarily by the noise level of the receiver and for most weather radars this is between 10^{-12} and 10^{-13} watts.

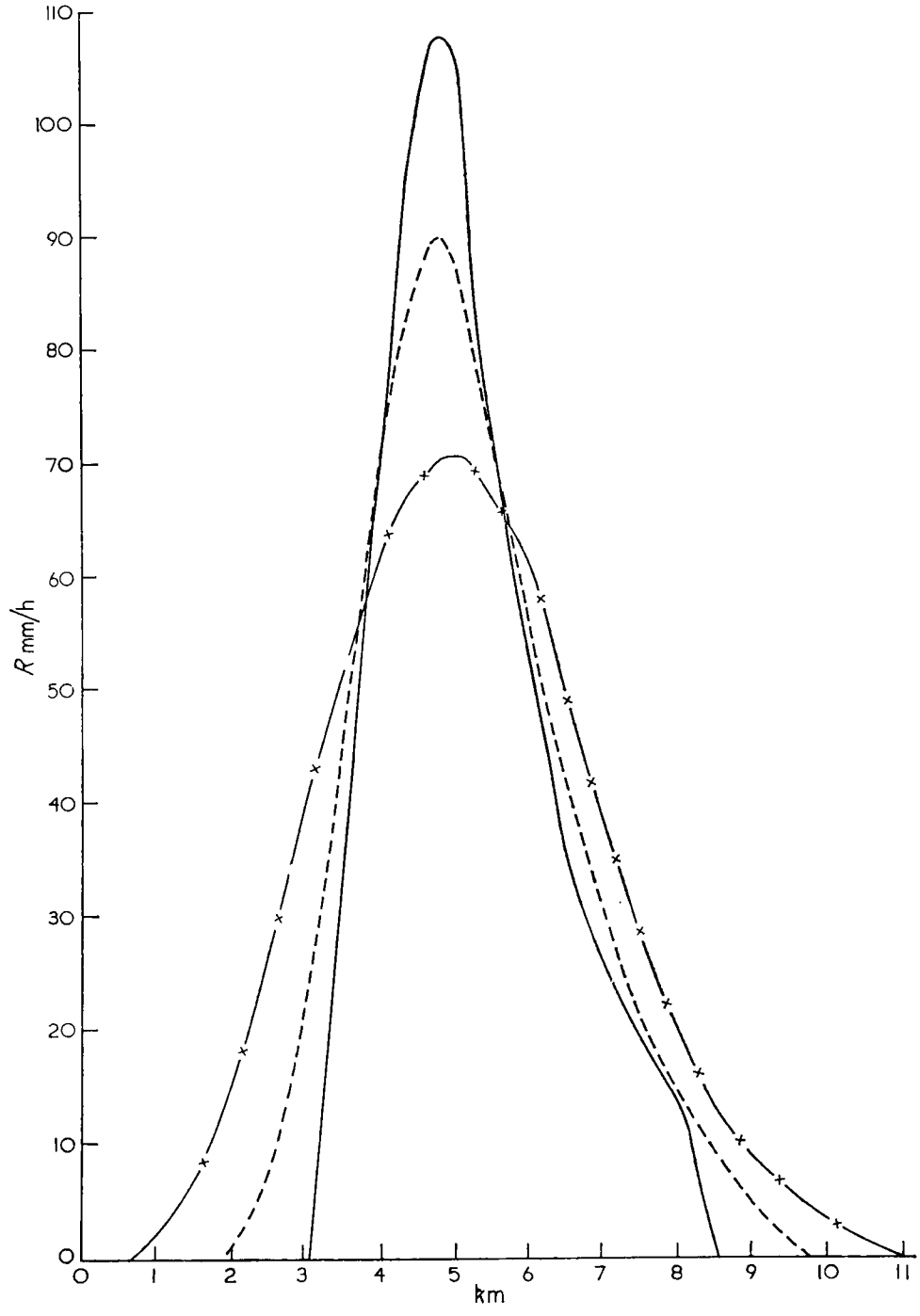


FIGURE 12. Apparent rate of rainfall across a shower scanned by beams 2·4 and 4·9 km wide.
..... Actual rate of rainfall, — — — Beam 2·4 km wide, x—x Beam 4·9 km wide.

Assuming $Z = 200R^{1.6}$ and knowing the radar parameters we can use the radar equation to calculate the minimum detectable rate as a function of range, provided that uniform precipitation fills the beam. For example if

$$\lambda = 10 \text{ cm}; \theta_1 = \phi_1 = 2°; \bar{P}_r = 10^{-12} \text{ W}; h = 3 \mu\text{s} = 900 \text{ m}$$

and

$$P_t = 500 \text{ kW}$$

then

$$R_{\text{MIN}} \simeq 1{\cdot}6 \times 10^{-3} r^{1{\cdot}25} \qquad \text{where } r \text{ is in km.}$$

So even at $r = 100$ km, $R_{\text{MIN}} = 0{\cdot}5$ mm/h, and only a small error will be introduced into a measurement of rainfall amount due to light precipitation not being detectable at the larger ranges. For heavy rainfalls this error will be negligible. This conclusion has been verified for the American WSR-57 radar (which has similar parameters to those assumed) which detects almost all occurrences of 1 mm/h out to 160 km (Wilson[31]). However, the lower reflectivity in the snow region may result in the signal falling to below the minimum detectable level. This would be reflected in Figures 8 and 9, where R_a/R_G would fall to below the minimum detectable level at larger ranges.

TABLE IX. *Mean apparent rate of rainfall across a shower*

Horizontal beam width	Approximate range at which the beam would be this width		Mean R from P to Q in Figure 6
	2° *beam*	4° *beam*	*mm/h*
0 (actual shower)			25·3
1·2 km	32 km	16 km	25·9
2·45 km	64 km	32 km	26·9
4·9 km	129 km	64 km	30·2

13. *Radar calibration and stability*

To measure rainfall quantitatively the radar must be accurately calibrated, that is the constant of equation (3) must be determined. The radar parameters, however, are not entirely stable. The stability is determined primarily by changes in the receiver sensitivity and the constancy of the power of the transmitted pulses. The overall stability is generally taken to be in the region of 1 or 2 dB (e.g. Austin and Williams[32]). If we take $\pm 1\frac{1}{2}$ dB as the uncertainty in received power due to this instability, this is equivalent to a mean error of 25 per cent in a single estimate of R. The error in an estimate of rainfall amount over a period of time would probably be less than this because of a tendency for the parameters to vary about a mean, so that successive errors of R would tend to cancel out. However, in order to make reliable rainfall measurements it is essential to reduce all errors as far as possible to a minimum. Therefore frequent calibration of the radar, of the order of every half hour, should be made. This calibration would consist of metering the transmitter output, and passing a test signal through the receiver, and should be a simple routine matter taking at the most five minutes.

It is a matter of conjecture whether future developments will lead to a significantly more stable radar.

14. *Accuracy of the radar equation*

In order to evaluate the received power from a theoretical radar equation and compare it with that actually observed, we require not only values of the parameters of the radar but also some knowledge of the nature of the echoing precipitation. While the range and dielectric constant present no problems, Z is more difficult to measure. A sample drop-size distribution obtained for example with filter papers may be inaccurate at large diameters. It has been more usual for an experimenter to measure R and convert this to a Z value using a Z–R relationship. It is immediately obvious that different experimental results are to be expected if different relationships are used.

Probert-Jones[2] has summarized the experimental data. Almost all experiments have shown that the slightly different forms of the equation used by the experimenters overestimated the power received from rain. The revised form of the equation derived by Probert-Jones, which has been used in this paper, reduces the discrepancy between theory and experiment. Table X, based on the Table I of Probert-Jones,[2] shows that even with the revised equation there is still a tendency to overestimate the power received by one or two dB. It should be noted that many of the experiments were at attenuating wavelengths, which may have contributed to the discrepancy. The effect in the Hooper and Kippax[33] experiment would be small since they used a vertically-pointing radar, and were therefore working at very short ranges.

TABLE X. *Comparison of theoretical and measured received powers in rain (after Probert-Jones[2])*

Author	Discrepancy dB	Wavelength cm
Hooper and Kippax[33]	+1½	3
Atlas[34]	+1	3
Hood[7]	+2	3
Hood	+1½	10
Austin and Williams[32]	+3½	10
Roberts[35]	+1½	3
Roberts	− ½	0·8

Also below 3 cm the Rayleigh approximation and the assumption of spherical drops begins to introduce small errors. But it is most unlikely that these effects can account for much of the difference actually found.

The reasons for this discrepancy between theory and experiment are not known, although a contributory cause is probably the decrease of echo intensity with height. It is an embarrassment to rainfall measurements since an empirical correction factor of about 2/3 must be included on the right-hand side of equation (3). Since this correction may possibly be a function of the type of radar, it would have to be determined experimentally as described in the first paragraph of this section before rainfall measurements could be made.

15. *Calibration of radar using a small rain-gauge network*

The accuracy of a rainfall measurement over an area would probably be significantly improved if measurements were at the same time made over a small rain-gauge network. This would enable a direct calibration between received power and rainfall, and would eliminate much of the uncertainty in the Z–R relationship, the radar equation, and radar

stability. (A single calibration gauge would be insufficient because of the uncertainties of the wind drift of the precipitation in the beam—see Section 16.) Such a method would of course depend on there being rain at the calibrating network and also assumes that the calibration, i.e., the Z–R relationship, holds over an area—which as we have seen in Section 1 seems likely. To avoid delay in the radar rainfall measurements, the rain-gauge network would have to be fully automatic, and it would have to be possible to 'interrogate' it from the radar site. An experimental programme would be needed to decide on the required size and density of this network. It is tentatively suggested that 5 to 10 gauges over about 10 km^2 would suffice, provided rain drift could be allowed for. The network must of course be clear of permanent echoes, but not at too large a range since as the range increases so the beam more frequently intersects the melting layer. The improvement in areal radar rainfall accuracy would have to be investigated experimentally, but it seems possible that the mean error over a period of time would be better than $\pm$ 15 per cent and perhaps better than 10 per cent over the range where the beam is entirely in the rain. This method could of course be applied only over land and not to a shipborne radar.

A possibility for the future might be to use rainfall measurements over a small area obtained by the attenuation method (Part II) to calibrate the radar and eliminate the uncertainty of the Z–R relationship. Short radio links at 8 mm may possibly be developed in the future and if sufficiently dense over a small area might be used to measure areal rainfall over that area. This rainfall could be used to calibrate a radar measuring echo intensity over a much larger area.

We see then that there are possibilities of improving the accuracy of a measurement of rainfall over land when the beam is entirely in the rain. There does not seem to be any way of reducing the additional mean error of about 25 per cent, which occurs when the beam intersects the melting layer.

16. *Some experimental measurements of rainfall using radar*

Experimental investigations of the accuracy of radar rainfall measurements have been made in several countries—the most thorough, up to the time of writing (July 1964), being that of the Illinois State Water Survey. Some of their more important conclusions will now be summarized (Mueller *et alii*[26]).

In one experiment they compared the received power over 250 km^2 with areal rain-gauge amounts and found considerable scatter in the results, especially for 1-minute totals. This scatter was mainly due to rain drift. Many of the factors discussed in the previous sections could also have contributed. The errors are largest with point rather than areal comparisons. It is for this reason that a point calibration is not considered sufficient to calibrate a radar for areal rainfall measurements (Section 15). The Illinois group used 3·2-cm radar, and the results over longer time intervals (three hours for example) were hampered by attenuation at times.

The Illinois group also measured radar rainfall over two areas of 85 km^2 and 250 km^2, to determine the suitability of different sized areas in a radar rainfall comparison. They concluded that the larger area was preferable owing to rain drift. There is probably an upper limit to the area size, above which the 'edge' errors due to rain drift can be neglected. This area would be the most suitable for rainfall measurements since it would combine the greatest accuracy with the best resolution.

In an experimental estimation of areal rainfall with radar it is necessary to adopt some standard of rainfall measurement as a basis for judging the reliability of the radar estimates.

A suitable method is to relate the radar rainfall estimates for the area to a network of rain-gauges on that area. Then one way of expressing the reliability of radar rainfall estimates is in terms of the accuracy obtained with rain-gauge networks of various gauge densities. This equivalent rain-gauge density of course varies greatly with rainfall type. The actual areal mean rainfall is unknown and has to be assumed to be the mean of the recorded rainfall in the total number of rain-gauges in the area. Therefore it is essential that a dense network be used. For example the Illinois network contained 50 gauges in 250 km^2.

Stout, Neill and Farnsworth[36] presented the results of an experiment in this form. They measured the radar rainfall in four thundery storms in Illinois. Table XI summarizes their results. The authors concluded that the measurement of areal thunderstorm type rainfall by radar was, in the least satisfactory case, as accurate as that obtained by a rain-gauge network of 1 gauge per 780 km^2 (but attenuation of the 3·2-cm radiation may have worsened the radar estimate in this case), while in other cases it was considerably more accurate.

TABLE XI. *Radar measurements of rainfall. Stout et alii*[36]

Type of rain	Period	Equivalent rain-gauge density	Radar estimate as percentage of rain-gauge estimate
Single, rapidly moving thunderstorm cell	27 minutes	1 per 90 km^2	71
Widespread, thundery	25 minutes	1 per 325 km^2	158
Rapidly moving squall line	44 minutes	1 per 780 km^2	41
Rapidly moving squall line	40 minutes	1 per 50 km^2	91

In a similar experiment in Japan, Kodaira and Aoyagi[11] and Aoyagi[37] describe measurements made on a day of widespread rain using a 3·2-cm radar, but with attenuation approximately corrected for. Their rain-gauge network consisted of 27 gauges in 638 km^2 and must be considered as rather sparse for this type of experiment. They found that a 10-minute radar measurement was statistically equivalent to a gauge density of about 1 per 200 km^2. Results were less conclusive over longer periods of time, but showed a general increase of accuracy. Over the four hours of observation the difference between the radar and 'best' estimate was only 3·6 per cent and approximately equivalent to the accuracy obtainable with a network density of 1 gauge per 80 km^2.

Both these experiments assumed a Z–R relationship. An indication of the accuracy which might be attained using a small calibrating network is given in an analysis (Harrold unpublished) of some reduced-gain PPI photographs taken at East Hill during an hour of very heavy localized thunderstorms over the Cardington rain-gauge network (16 gauges over 16 km^2). The first 40 minutes of the rain were used to calibrate the radar and this calibration was used to estimate the radar rainfall in the remaining 20 minutes. The mean radar rate of rainfall over the area was $5\frac{1}{2}$ mm/h compared with the 'best' estimate of $7\frac{1}{2}$ mm/h—an error of about 25 per cent, but the rainfall was of such a localized nature that this accuracy was better than that obtainable with a rain-gauge density of 1 per 10 km^2.

These experiments only indicate the order of accuracy of a radar rainfall estimate and firm conclusions cannot be drawn. The equivalent rain-gauge density for a radar measurement is a function of the type of rain—whether widespread and uniform or localized, and the period of time over which measurements are made. A tentative conclusion is that with

a radar having similar parameters to those suggested in the preceding sections, an accuracy of better than that of 1 gauge per 100 km^2 should be obtained over periods of longer than $\frac{1}{2}$ hour except in uniform rains. In spite of the many years over which the importance of experimentally assessing the accuracy of radar rainfall measurements has been realized, no complete experimental project seems to have been reported (July 1964). Only a very limited amount of work has been done in this country.

The possibility has been considered of measuring radar rainfall over the Winchcombe rain-gauge network in the Cotswolds, about 20 km from Malvern. It has been shown that the radar should be about equivalent in accuracy to 1 gauge per 100 km^2, so that to compare radar and rain-gauge rainfalls a dense network (1 gauge in about 5 km^2) over about 100 km^2 is desirable. If a smaller network is used then the percentage error in the radar measurement over this and smaller areas can be found, but it would not be possible to determine the equivalent rain-gauge density. This applies to measurements over the Winchcombe network, which extends over about 15 km^2.

PART IB—INSTRUMENTAL DEVELOPMENTS

17. *Aims*

It has been shown that a radar can be used to measure rainfall over an area. A practical working system must be fully automatic and should not require either a research scientist or a full-time technician in everyday operation. Instrumental developments must aim at automatically providing a direct display of accrued rainfall since the time of reset over specific areas of interest. Cost will be an important aspect.

There are two lines of approach. In one, the time integration of the radar echo presented on a PPI display would be achieved photographically, in the other, this integration would be performed electronically. The following paragraphs are intended to show in general terms the equipment that might be needed to achieve a useful integration of areal rainfall, and do not give a complete description of the circuitry needed. The progress reported by other workers toward a practical system will be briefly reviewed, and some broad ideas for future consideration will be presented.

18. *Correction for range effect*

The power received from precipitation filling the beam is a function of range (equation (1)) as a result of the divergence of the radar beam. In order to make a quantitative estimate of precipitation it is necessary that the signals be corrected for this range effect. This has been done successfully by several experimenters and no difficulty is to be expected in what are mostly well-known techniques so only a brief survey will be given here.

Katz[38] claims that a graded optical filter over the display such that light from the centre is cut down relative to larger ranges can provide quantitative correction.

Kodaira[23,39,40,41] has built successful circuitry in which the received signal is converted to logarithmic form in a logarithmic amplifier (see Section 20) and a voltage proportional to log r added to it. This appears to be the best procedure in a radar rainfall measurement.

Kodaira's method has the additional advantage that atmospheric attenuation can also be corrected for with very little additional equipment. At 10 cm however, $k = 0{\cdot}007$ dB/km (Gunn and East[19]) and no serious error is involved if this is neglected.

Ray[42] made the gain of a logarithmic amplifier a function of range. However, the voltage required to do this is not a simple mathematical function and its production does not lend itself well to conventional electronic circuitry.

The Stormy Weather Group at McGill University have applied range correction by altering the gain control of the pre-amplifier. Legg[43] showed that the correction was not perfect but expected that improvement could be attained fairly simply. If so this method could provide a simple alternative to Kodaira's method.

19. *Dynamic range of a conventional radar*

Values of the reflectivity factor Z vary from about 10 mm^6/m^3 in light rain to over 10^6 mm^6/m^3 in very heavy rain, so that at any range the power received at the aerial will vary over 50 dB. A conventional radar receiver and display are not quantitative over such a range. Both large and small signals are compressed (e.g. Reintjes and Coate,[44] Legg[43]) in the receiver, and strong signals are purposely further compressed between the receiver and display, to facilitate transmission over a cable between the two. A cathode-ray tube is only linear for at most a 25 dB range of input power. Other factors affecting the quantitative nature of the display are (*a*) the fact that the minimum luminance of an area on the display depends on the luminance of other areas of the screen (e.g. Haines[45]), (*b*) the persistence of the echo on the screen, and (*c*) changes of the tube characteristic as it ages.

The combined effect of these limitations on the signal which can be quantitatively handled results in the overall dynamic range being less than 20 dB of received power. Legg for example quotes 16 dB. As it stands such a system cannot be used for quantitative precipitation measurements.

The requirements of a system to be used for the extraction of quantitative data from the received signals differ somewhat depending on whether the precipitation falling over an area is to be measured by photographic or electronic means. If photography is to be used then both the cathode-ray tube and photographic processes, as well as the receiver, must be quantitative, whereas if electronic integration is used this can be performed at the output of the radar receiver, obviating the actual display and photography.

20. *Increasing the dynamic range of a receiver*

The dynamic range of almost all linear receivers is less than the 50 dB we require for rainfall measurements. However, such a receiver can, with difficulty, be developed. For example the dynamic range of the linear receiver of the mobile Doppler radar being developed at The Royal Radar Establishment, Malvern is 55dB.

For many years the technique of stepping the receiver gain has been used to extend the dynamic range of conventional radars. The technique is simple but a disadvantage is that it takes time to do a reduced-gain run. Also it is wasteful because all the information that can be displayed by stepping the gain over five or more scans is already contained in the signals reaching the antenna in a single scan.

A logarithmic amplifier is a more convenient method of extending the dynamic range of the receiver for rainfall measurements. For such an amplifier the output voltage is $V = A \ln(V_r/B) + C$ where A, B and C are constants. Logarithmic amplifiers have been used by several experimenters and are standard components of some recent weather radars (e.g. the WSR-57). Both Kodaira[39] and Legg[43] obtained dynamic ranges of over 50 dB.

21. *Quantitative displays*

Perhaps the most quantitative display that has been developed is the 'Grey Scale' at McGill University (Legg,[43,46] Barath[47]). The precipitation echo is displayed on a PPI in shades of grey depending on the signal strength. The response of the human eye is such

that equal increments of log luminance are equally discernible (Blackwell[48]) and the aim of the display has been to arrange that these should represent equal increments of log R.

A logarithmic amplifier provides sufficient dynamic range. If the output V of this amplifier is applied to the display tube a plot of log luminance against log R is not linear because of the power-law response of the cathode-ray tube. The more intense signals are compressed into a small region of discernible tones. To overcome this a gain control and exponential amplifier are added.

Ideally the output of the log amplifier is

$$V = V_1 \ln \frac{P_r}{P_1}, \text{ where } P_1,\ V_1 \text{ are constants.}$$

The output of the gain control is

$$V' = GV, \text{ where } G \text{ is the voltage gain}$$

and the output of the exponential amplifier is

$$V'' = E \exp(F V') = E \exp G\, V_1\, F \ln \frac{P_r}{P_1} = E\left(\frac{P_r}{P_1}\right)^{GV_1F}, \text{ where } E \text{ and } F \text{ are constants of the exponential amplifier,}$$

Now if this voltage is applied to a cathode-ray tube having a power-law exponent n the overall response of the radar will be

$$L = L_0 (V'')^n = (L_0\, E^n\, P_1{}^{-GV_1Fn}) P_r{}^{GV_1Fn}$$

or

$L \propto P_r{}^{\gamma_r} \propto R^{1{\cdot}6\gamma_r}$ (provided that P_r is averaged over a number of pulses—see Section 23).

where $\gamma_r = G\ V_1\ F\ n$ and is termed (by Legg) the "radar gamma" since it has analogies with the film γ in photography. So $\log L \propto \log R$, which is the condition aimed at in the display. With this adjustable system, G being a control, a chosen range of signal power can be fitted into the limited range of luminance, which amounts to controlling the overall dynamic range of the radar.

Range correction is included in the system (Section 18). A neutral density optical filter is used to correct for the variation in the spacing of trace lines (Legg[43] p. 65). The background luminance is suppressed (Barath[47]).

Hence the Stormy Weather Group have developed a radar display which quantitatively shows rainfall rate. Marshall (see Legg[43] p. 101) has suggested that the equipment may be used for the quantitative integration of rainfall. He states that the radar gamma can be adjusted so that $L \propto R$ and the display could then be integrated either (*a*) photographically, or (*b*) with a photocell placed in front of the display tube, as follows:

(*a*) Rainfall could be measured from the transmission through a positive transparency, the transmission being proportional to the rainfall. By masking an area the total rainfall in that area would be given by a simple measurement of the transmitted light with a photocell. Thus, suppose a negative film is exposed during many radar scans. The transmission T_P of the positive picture will provide a good measure of the rainfall integrated over time if

$$T_P \propto [R(t_1) + \ldots R(t_m) + \ldots R(t_n)]^{\gamma_N \gamma_P} \tag{5}$$

where $R(t_m)$ is the rainfall rate at time t_m, and γ_N and γ_P are the gammas of the negative and positive films.

In practice, assuming reciprocity of the film

$$T_P \propto [E(t_1) + \ldots E(t_m) + \ldots E(t_n)]^{\gamma_N \gamma_P} \tag{6}$$

where $E(t_m)$ is the exposure of the negative resulting from $R(t_m)$. So (5) is achieved in general only if $E(t_m) = \text{const.}\ R(t_m)$, which will be so when $L \propto R$.

Only if $\gamma_N \gamma_P = 1$ could T_P be directly proportional to rainfall. Marshall does not think this is easy to achieve in practice. The types of film would have to be carefully selected and the processing carefully controlled. This is a definite weakness and at the present time it seems unlikely that this method could be other than a research project in measuring rainfall. The difficulties are too great for the method to be of practical value, at least in the near future.

(*b*) A photomultiplier tube placed in front of the display would give an output signal proportional to L, and the integration could be performed over an area required. Integrating the output signal over a period of time would give the accrued rainfall and this could be indicated on a counter. Provided the photomultiplier can be made accurately quantitative and the display is also quantitative, this may be developed into a practical method of rainfall measurement in the future.

22. *Quantitative displays versus electronic integration*

This is a suitable stage to compare the advantages of the two possible methods of integration. Any photographic or photomultiplier method depends on a quantitative display and the grey scale appears to be the best approach to this to date. In the photographic method the receiver, display and photographic processes must be fully quantitative. Some difficulties in attaining this have already been noted. Others of importance are the persistence of the display and the change in the characteristic of the tube as it ages. The rainfall amount is not available immediately because of the additional stages of processing the film and analysing it for the rainfall data. An advantage is that details within an area are retained. Compared with this the electronic method appears to the author to be simpler since only the receiver must be quantitative. If the output of the receiver is directly relatable to the rate of rainfall then electronic integration can be performed and the accrued rainfall indicated on a counter. *A priori* there appear to be fewer sources of error in the electronic method and the remainder of this part of the paper will concentrate on its possibilities.

23. *Averaging of precipitation echo*

The echo return from rain is incoherent. The random fluctuations about the average value limit the accuracy of the information obtainable. Marshall and Hitschfeld[49] have shown that instantaneous echo measurements have little meaning.

In equation (1) $\overline{P_r}$ is defined as the average of the instantaneous power P_r over a sufficient number of pulses to ensure an adequate statistical sample. In practice averaging j independent intensity values will reduce the standard deviation of the measurements by a factor $1/\sqrt{j}$. In general the number of pulses integrated, n, does not equal j, the number of statistically independent samples, the relation between them being a function of the pulse length, pulse repetition frequency and band width (see for example Kodaira[40]). In practice, depending on the radar used, a value of n between 10 and 50 is usually considered statistically adequate in measuring $\overline{P_r}$.

The necessity of integrating the rain echo will be mentioned further in the following sections.

24. *Automatic electronic integration of radar rainfall*

There are three basic methods of instrumenting the radar to measure rainfall.

(*a*) Integration of the received voltage directly over the required area and time.
(*b*) Use of a stepped gain or levelling method so that the signals are made to operate a counter when they exceed known thresholds (digital method).
(*c*) Conversion of the input voltages to voltages proportional to R by an analogue method and summation of these voltages.

The basic requirements of these methods will be outlined in the following three sections.

25. *Measurement of input voltage direct*

We have seen (equation 3) that

$$\bar{P}_r \propto \frac{\text{const } R^{1\cdot6}}{r^2}$$

or

$$\bar{V}_i = \text{mean input voltage} \propto \text{const } \frac{R^{0\cdot8}}{r}.$$

Let the range-corrected voltage be

$$\bar{V}_r = \text{const } R^{0\cdot8} = CR^{0\cdot8} \tag{7}$$

If we sum these voltages we are in effect summing values of $R^{0\cdot8}$ or alternatively we are assuming $Z \propto R^2$. Consequently this method cannot be expected to produce precise rainfall totals. If we sum $\bar{V}_r$ i.e. find $C \Sigma R^{0\cdot8}$, then the best estimate of rainfall will be $(\overline{R^{0\cdot8}})^{1\cdot25}$. This can only equal $\bar{R}$ at a constant rainfall rate.

Table XII shows examples of values of R which might occur in practice and compares $(\overline{R^{0\cdot8}})^{1\cdot25}$ (column 3) with $\bar{R}$, (column 6). The values of R in this table can be pictured as occurring either over equal small areas within a larger area, or over small intervals of time within any one area. Example 1 would be representative of widespread frontal rain in this country, example 2 represents a heavy rain cell passing over an area, or an instantaneous distribution over an area over which measurements are being made, and examples 3 and 4 represent a heavy shower localized in time or space. It is seen that in the worst of these cases (3), the error in the rainfall estimate, excluding the errors discussed earlier, would be less than 100 per cent.

In fact it is found that the error in assuming that $(\overline{R^{0\cdot8}})^{1\cdot25}$ equals R is small when the variations of R are small. The error is greatest when there are a large number of values of $R = 0$ contributing to the final mean rate of rainfall. The accuracy of the method can be considerably improved if we can minimize the integrating time over which $R = 0$. This entails either measuring over a small area (say less than 10 km × 10 km) so that the rain will generally fill the area, or selecting the area of measurement. (It would in fact be useful in any rainfall measurement if we could select the area(s) over which to measure rather than have fixed areas.)

An area selector unit has been suggested by Kodaira.[23] This unit contains its own PPI with a mask over all but the area of interest. A photo-tube in front of this PPI has an output only during the time that the sweep is in the area, and this output switches on the receiver. The effectiveness of this system is not known, but several difficulties can be envisaged. For instance the time a single pulse is within the area is very short and so the

switching process must be very rapid. Persistence of the sweep spot on the screen must also be negligible. An alternative method of selecting the area over which measurements are to be made is to use adjustable range gates. In effect it is required to select two ranges and two azimuths, within which measurements are to be made. All four boundaries of the area would have to be adjustable.

Integrating voltages over an area and period of time should not provide any major electronic difficulties and the cost of equipment should be comparatively low. The integrated voltage would be displayed on a counter. This voltage is proportional to $\overline{R^{0\cdot8}}$ and the observer can calculate the best rainfall estimate $(\overline{R^{0\cdot8}})^{1\cdot25}$. Alternatively with additional circuitry, this stage might also be made automatic.

The nearest approach to this type of integration known by the author to have been published is in a paper by Austin and Richardson.[50] Previously a pulse integrator had been developed by the M.I.T. Radiation Laboratory (Goldstein[51]) and adapted by the M.I.T. Weather Radar Research project (Williams,[52] Ligda[53]). The instrument measures and records the average peak amplitude of a number of video pulses at one range, by utilizing a storage circuit in which a capacitor is charging at a high rate during the pulse and discharges very slowly between pulses. It has been pointed out (e.g. Austin and Williams[32]) that the instrument measures $\overline{V}$, and $\overline{P_r} = \overline{V^2}$. Assuming a very large number of scatterers and that their positions are random

$$(\overline{V})^2 = 0{\cdot}79\,\overline{P_r}\ ,$$

therefore

$$\overline{V_r} = 0{\cdot}89\,CR^{0\cdot8}$$

from equation (7). This effect must be included in the computation of R. It applies to any component computing a mean voltage for a rain echo. The circuitry was developed by Austin and Richardson to integrate the echo over an area rather than at a point. The mean over an area 16 km $\times$ 10° was found. A scanning rate of one revolution in four minutes was used so that the mean over the area was found in about seven seconds. Using this one integrator they measured over all areas in the range interval 20–100 km so that measurements were only repeated every half hour in any one area. Consequently their system was not fully practical—a practical system would require either a whole series of integrators or, probably more simple, a device to switch the integrator to a different counter for each area. Austin and Richardson did not have an adequate rain-gauge network with which to check their measurements. Consequently the device was not adequately proven.

We have assumed in the above paragraphs that the receiver is linear. It was stated in Section 20 that linear receivers can with difficulty be developed with sufficient dynamic range. However, this range is usually attained with a logarithmic receiver. In this case the video pulses being integrated would be proportional to $\overline{\log V_r}$ or $\overline{\log R^{0\cdot8}}$. A statistical analysis of the effect of integrating logarithmic rain echoes has been developed by Kodaira.[41] It will not be repeated here. The best estimate of rainfall is then antilog $(\overline{\log R})$, and examples of this value are also shown for the rates of rainfall in Table XII. It is seen that when the rain is very unevenly distributed the error is larger than that due to the assumption that $(\overline{R^{0\cdot8}})^{1\cdot25}$ equals $\bar{R}$. This is especially so when there are many values of $R = 0$ included in the mean. Consequently if the video pulses are to be averaged to estimate rainfall, a linear receiver is preferable to a logarithmic receiver.

Summing up this section, the video pulses from a linear receiver with sufficient dynamic range can be integrated to provide an areal measurement of rainfall over a period of time.

However, the computation is not exact since it assumes $Z \propto R^2$, whereas we have seen that a better relation is $Z \propto R^{1.6}$.

TABLE XII. *Examples of* $(\overline{R^{0.8}})^{1.25}$, $\overline{R_{\text{LEVELS}}}$ *and* antilog $(\overline{\log R})$

Example No.	Successive values of R mm/h	$(\overline{R^{0.8}})^{1.25}$	$\overline{R_{\text{LEVELS}}}$	antilog $(\overline{\log R})$	$\bar{R}$
1	$\frac{1}{2}\ \frac{1}{2}\ \frac{1}{2}\ \frac{3}{4}\ 1\ 2\ 1\ \frac{3}{4}\ \frac{1}{2}\ \frac{1}{2}\ \frac{1}{2}\ 0$	0·67	0·78	0·59	0·7
2	9 20 40 60 80 100 80 60 40 20	41·7	48·0	48·0	43·6
3	12 units as (2) plus 88 units of $R = 0$	2·9	5·58	0·20	5·23
4	12 units as (2) plus 88 units of $R = \frac{1}{2}$	3·7	6·07	0·82	5·70

26. *Measurements based on levelling methods*

Several methods based on measuring signals above known levels have been developed and three of these will be described here. In principle a pulse activates a counter if it is above a known level, and does not activate the counter if it is below this level. The level is changed before each sweep of the aerial. Since these levels are finite steps apart, details of the signal between them is lost. Examples of the $R_{\overline{\text{LEVELS}}}$ are included in column 4 of Table XII. In this calculation the levels are assumed to be at $\frac{1}{2}$, 1, 2, 4, 8, . . . mm/h. A signal falling between two levels is assumed to equal the mean rate of rainfall between those levels. A signal falling on a level is assumed to be half below and half above that level. In the examples the error resulting from approximating the signals to discrete mid-level values is not large and in particular is significantly less than the error in assuming $(\overline{R^{0.8}})^{1.25}$ equal to the mean rate of rainfall when there are cases of $R = 0$.

The Illinois State Water Survey have developed an area integrator (Farnsworth and Mueller[54]) and (Mueller[55]). This instrument measures the area of echo above an adjustable threshold over a fixed selected region of area 256 square km. So, unlike the Pulse Integrator of Austin and Richardson, this instrument measures an area not an echo intensity. The video signal was averaged in time along a radial. Although this is not the type of averaging of P_r required by theory, it does help to reduce the effect of the incoherent signal on the display. It was admitted that a better method of power averaging was necessary. This integrated video signal operates a gating circuit which allows pulses to pass to a counter, so that the number of pulses counted is proportional to the area of precipitation above the threshold. Use at different gain settings provides information on the echo area above various levels. The area obtained by the area integrator for each of the individual step levels can be multiplied by the theoretical rainfall intensity for each level, and the values can then be summed to determine the total amount of rainfall. However, this was not done automatically at the time of the 1958 paper, which states that a desk calculator was used in the computations.

Given an adequate 'memory' this instrument could presumably be made automatic to provide rainfall information. Several areas could also be dealt with. However, the method does not seem to have been developed further in the last five years.

Kodaira[56,57] has built an areal rainfall integrator. The basic radar receiver used includes features developed successfully in earlier equipments. The dynamic range is increased by a logarithmic receiver, range correction is applied and levelling circuitry is included. The

'levelled' range-corrected signals are converted from PPI scanning video signals to television scanning audio frequency signals by using a storage tube. This storage tube also integrates signals over a number of pulses, thus removing much of the noise-like fluctuation of the precipitation echo. The levels of the signal correspond to about twofold intervals of rainfall intensity (assuming $Z \propto R^{1.6}$). Column 4 of Table XII indicates the accuracy which could be expected.

A digital method, the counting of clock pulses, is used in the calculation of the rainfall. The storage-tube area is divided into 60 complete grid squares, each about 10 km × 10 km, and in each square there are $22 \times 22 = 484$ sampling points. A separate binary counter is used for each grid square. When a signal occurs at a sampling point a clock pulse is registered in the binary counter appropriate to that square. For higher levels of signal intensity higher stages of the binary counter are used, since the rainfall levels differ by a factor of two. This is followed by binary to decimal conversion. The resulting figure is proportional to areal rainfall. An antenna rotation takes 6 seconds, the level-switching takes place at 12-second intervals and one sequence of nine levels is completed in 108 seconds, and is repeated a minute later.

A point to note is that this method does not appear to be confined to the radar receiver used by Kodaira. The basic requirement is that signals above a series of threshold levels are fed into a binary counter system, the levels being separated by a factor of two in R.

The grid areas over which measurements are to be made are selected by inserting plugs into the appropriate part of a selector panel. Rainfall amounts are displayed directly in millimetres. The instrumentation did not appear to have been tested over a rain-gauge network at the time of Kodaira's latest paper (1963).

Collis[58] has also developed a 'radar precipitation integrator' using a stepped-gain approach and a digital computing system. The data are presented on a display consisting of a series of electro-mechanical counters arranged in a grid set in a map showing the area covered by the radar. The counters record directly, in inches, the rainfall accrued at each point since the time of reset.

In a prototype equipment measurements are made over 141 'points' in a circle 200 nautical miles in diameter. Each sampling 'point' is an area 4 n. miles by 1°. Consequently to this extent the system is not an areal rainfall integrator but a multi-point sampling device. The length and width of the range gate are adjustable, but the method of data acquisition (see next paragraph) precludes a true areal measurement.

Range-corrected video signals from the logarithmic receiver of a WSR-57 together with trigger and antenna position data, are processed in the integrator in the following way. The location of each of the address points of the grid at which rainfall is to be integrated is so arranged that only one point lies on any one radial from the radar site. So provided the range of each address can be established, it may be identified by reference to the azimuth of the radar antenna. The intensity of the echo at any address is reported as a series of up to six pulses, corresponding to the six intensity levels. To enable the data to be processed more slowly than they are acquired in real time a punched-tape buffer store is used, which takes advantage of a 10-minute gap between observation cycles. Class levels may have a non-linear relationship, since any 'reasonable' weighting factors can be applied in the photomultiplier unit.

However, it would be difficult to convert this point measuring device into an areal rainfall integrator. Collis[58] states that "the object of the development was clear cut—to provide a direct display of accrued rainfall since the time of reset for operational applications in

flood forecasting and river control. The essentially practical nature of this objective is a controlling factor in the approach, which is designed to exploit the instantaneity and convenience of radar observations at the expense of any preoccupation with great precision". However, the nature of the sampling device is such that it could be inadequate in localized rain. It should be adequate in widespread heavy rain but could fail to measure rains heavy enough to produce flash floods since in effect each sampling area of 4 n. miles by 1° has to be assumed to be representative of an area of about 200 square n. miles. The instrumentation has not yet been adequately tested over a rain-gauge network.

Of these three methods of measuring rainfall using a stepped-gain or levelling approach, Kodaira's appears to be the most useful and also that which would be of most value in a practical operational use.

27. *Analogue computing methods*

The basic idea in computing rainfall by an analogue method is to convert the voltage in the receiver to one proportional to R, and then to sum these voltages. Provided no errors occur in the computer this is the most exact method of measuring the rainfall, the errors being limited to those discussed in Part 1.

Kodaira[23] developed an analogue computing system for measuring rainfall over an area. The receiver dynamic range was increased using a logarithmic amplifier and range correction was applied. Since a 3·2-cm radar was used, rain attenuation was also allowed for but will not be considered here since possible correction methods have been discussed earlier. A block diagram of the relevant part of the computer is shown in Figure 13.

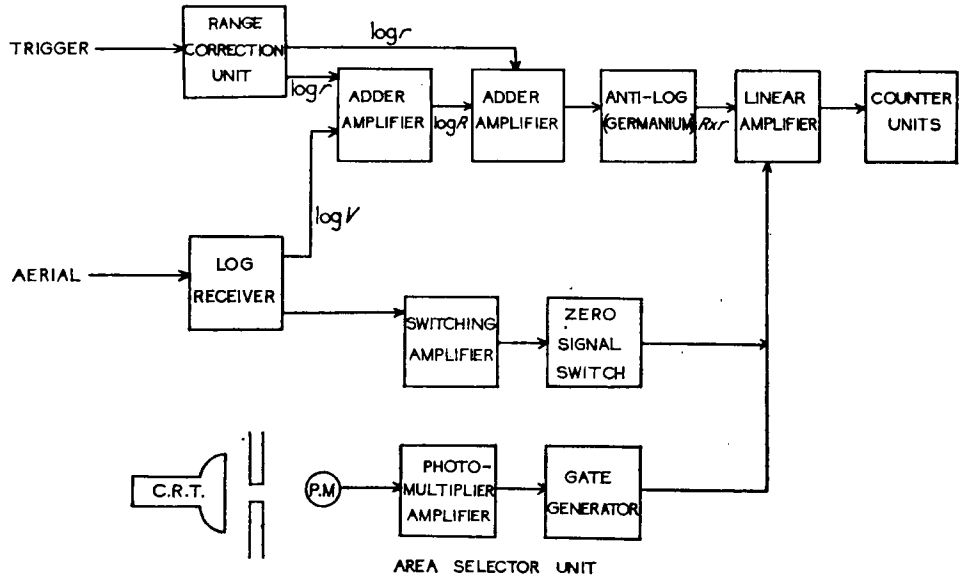


FIGURE 13. Simplified block diagram of analogue computer for rainfall integration, after Kodaira.[23]

The computer includes a discharge circuit to integrate the rainfall over a selected area for a period of time. A second adder is included to allow for the increasing area covered by the pulse with range, the area being about proportional to the range. (A unit of this type must be included in any rainfall computation over a large area using PPI scan.) Selection of the area over which to integrate is made by masking the face of a cathode ray tube.

The main limitation of the system was that the dynamic range of the antilog circuit was only 32 dB of input voltage. This limited the values of R and r which could be handled. Kodaira set these limits at 10 km mm/h and 400 km mm/h. He hoped this range would be increased to 60 dB in a second model, but the author has seen no report of this actually being achieved. One very inadequate test of the computer was mentioned in Kodaira's paper. On a day of widespread light rain the mean rate of rainfall over half an hour in a ring extending from 10 km to 30 km range was computed to be 0·4 mm/h. One rain-gauge near the radar recorded 0·8 mm/h for this period.

This analogue computing system is also fitted to another Japanese Weather Radar (Oki Electric Industry Co., Ltd[59]) and according to Kodaira (private communication) operates well. The radar can measure rainfall over any area out to 50 km.

The limitations of the antilog circuits may have been decreased since the time of Kodaira's paper in 1955. However, the grey-scale circuitry discussed earlier can perhaps be modified for use in an electronic integrating device, although this does not appear to have been suggested in any of the Stormy Weather Group reports.

We have seen that the output of the exponential amplifier is

$$V'' = E\left(\frac{\overline{P_r}}{P_1}\right)^{GV_1F} \propto R^{1\cdot6GV_1F}$$

and presumably G can be adjusted to make $1\cdot6\ GV_1\ F = 1$. The voltage output of the receiver is then directly proportional to R. These voltages can then be integrated in any one of a number of ways—viz. using a discharge circuit, storage tube, delay line or magnetic tape or drum. It will be preferable if the noise-like fluctuations of the rain echo are integrated out early in the system, as assumed in the last equation. If the integration is performed on V'' then a statistical correction will have to be calculated. It has been shown that

$$\overline{P_r} \propto R^{1\cdot6} \text{ and } V'' \propto \overline{P_r}^{GV_1F}$$

$$\text{therefore } \overline{V''} \propto \overline{P_r^{GV_1F}}$$

if the integration is performed on V''. Therefore it is required to establish a relationship between

$$\overline{P_r^{GVF}} \text{ and } \overline{P_r}^{1/1\cdot6}$$

in order to relate $\overline{V''}$ and R.

28. *Frequency of observations*

It is preferable that the rainfall measurements should not take all of the operating time of the radar. Bigler and Tarble[60] found that an interval of five minutes between observations over an area does not appreciably reduce the correlation between integrated echo and actual rainfall, in Texas. Consequently, areal rainfall measurements can be made at five-minute intervals, leaving the radar free for other uses in between. In less extreme rainfall regions a 10-minute interval may well prove adequate.

29. *Present-day practical radar rainfall measurements in the U.S.A.*

In the U.S.A. the ultimate aim of radar rainfall research is to develop instrumentation to measure automatically rainfall over an area. The circuitry has to be complex and is only just reaching the stage of being perfected. To fill the gap before this development

other more approximate methods have employed photography as the integrating tool in routine work.

Conover and Hiser quote Rockney as the first to use photographic integrations of the PPI echoes to estimate rainfall. The study was continued in a project at the University of Miami (Hiser *et alii*[61]), the film being exposed continuously for the entire period of integration of from one to six hours. The resulting density of the film as measured by a densitometer was compared with the precipitation measured by rain-gauges. A disadvantage of this method is that the radar is in continuous use and cannot be employed in other investigations. We have already stated that Bigler and Tarble found that an interval of five minutes between exposures can be tolerated. This latter method of multiple exposures is in use in the United States Weather Bureau in flood prediction, e.g. McCallister,[62] Leber *et alii*,[63] and Flanders,[64] but in a secondary role, as a supplement to the existing rain-gauge network.

The procedure is to make successive fixed reduced-gain exposures on the same Polaroid transparency over a period of an hour. The resulting photograph only shows areas of persistent and/or intense echo but this is stated to be of definite practical value. For example, Leber *et alii* using multiple-exposure transparencies and information obtained from reduced-gain runs of 12 3-dB steps made every 20 minutes, constructed a 24-hour radar rainfall map which agreed very well with that drawn from rain-gauge information.

A radar transponder rain-gauge has also been developed (Flanders[64]) with signal increments of (say) 0·1 of an inch when triggered by a remote WSR-57. This can serve to verify the rainfall pattern on a multiple-exposure transparency by giving an actual rainfall total.

PART IC—CONCLUSIONS AND RECOMMENDATIONS

30. We have shown that with a suitable radar it should be possible to measure rainfall quantitatively over an area as it falls. A wavelength of 5 cm or more is necessary in Great Britain, and where heavier rains occur 10 cm must be used. The beam should be 2° wide or less in the vertical to ensure measurements over sufficient range, and preferably 2° wide in the horizontal also, although measurements may be possible to about 60 km with a horizontal beam width of 4° if an additional error of about 20 per cent can be tolerated. The beam should be used at a low elevation angle in order that measurements can be made over as large a range as possible before the melting level is intersected. The siting of the radar will be of importance if permanent echoes are to be minimized, but even at a very good site measurements will probably not be possible within the first 15 km unless CAPPI circuitry is incorporated. Side lobes should preferably be more than 20 dB below the main lobe if inaccuracies in the measurement of localized rains are to be minimized.

When the beam is entirely in the rain a mean error of about 25 per cent in a measurement of rainfall from a storm is to be expected because of variations in the relationship between Z and R, unless a small calibrating network is used. The accuracy is less when the radar beam intersects the melting level and passes into the snow region. No experimental results have been reported on this topic, but the mean error may, very approximately, double. There does not appear to be any simple method of reducing this error.

Measurements can best be made over an area, and the integrations performed over at least an hour. Measurements need not be made continuously. American work suggests an interval of 5 to 10 minutes between observations.

Of the three basic instrumental developments the analogue method should be the most accurate and for this reason is the preferable method, but it is also probably the most

expensive. The digital method is more accurate than integrating the incoming voltages directly but the latter has the advantage of probably being the simplest to develop. Its accuracy is improved if the area over which measurements are made can be selected.

In order to determine the accuracy of radar rainfall measurements we have seen that the rainfall total can be compared with rain-gauge readings. Experiments in other countries show that the density of the equivalent rain-gauge network is of the order of one gauge per 100 km^2.

The only radars known to the author which have the required parameters for measuring areal rainfall accurately are the American WSR-57 and FPS-10, both 10-cm radars, and recent Japanese radars at 5·66 cm. The recently designed Decca Type 43S, also a 10-cm radar, should also meet the requirements. However, limited rainfall information can be obtained using radars not specifically designed for quantitative weather observations. The Meteorological Office Type 42A weather radars are at 3·2 cm and we have shown that this wavelength is less suitable for quantitative measurements because of attenuation. After nearly a decade of research, reliable automatic attenuation correction has not been developed. The dynamic range of the receiver of this radar is also less than required to avoid saturation in heavy rains—so that a stepped-gain approach to measuring rainfall would be necessary.

Summing up, it is felt that the accuracy attainable using a radar, together with the advantages of instantaneous measurements over many hundreds of square kilometres, makes this a worthwhile practical method of measuring rainfall, especially in more remote river catchment areas, over the sea, and in climates where the melting level does not interfere with the accuracy. It is less useful in this country in winter because of the low melting level. The accuracy attainable in this country has not been verified experimentally and instrumental developments in other parts of the world are still in the research stage. Both these topics could be usefully be investigated further.

Part II. Measurement of Rainfall based on Attenuation

1. *Basic equations*

The basis of this method of rainfall estimation is to measure the attenuation of a signal as it traverses a known path length of precipitation, and to relate this attenuation to the mean rate of rainfall along the path. Using the notation and units given at the beginning of this paper

$$k = 217 \int_0^\infty N_D Q(D\lambda)\, \mathrm{d}D \ \text{dB/km} \quad . \tag{8}$$

The Mie theory of scattering shows that for a spherical drop

$$Q(D,\lambda) = \frac{\lambda^2}{2\pi} - \mathrm{Re}\left(\sum_{n=1}^{\infty} (2n+1)(a_n + b_n) \right) \tag{9}$$

where a_n and b_n are the coefficients of the nth magnetic and electric modes and are made up of spherical Bessel functions with arguments $\pi D/\lambda$ and m. Values of $Q(D, \lambda)$ have been tabulated for spherical raindrops in the microwave region by Ryde and Ryde.[65]

Equation (8) shows that k is a function of N_D. Since there is no unique drop-size distribution at any given rainfall rate there is no unique relationship between k and R, but empirical relationships can be found.

2. *k–R relationship*

(*a*) *Theoretical.*—Both k and R are calculable from a drop-size distribution. Such calculations have been performed by Ryde and Ryde[65] and by Haddock,[66] assuming a Laws and Parsons distribution. Haddock's results can be approximated by the relationships already given in Table IV. This table shows that above about 3 cm the attenuation is not sufficient to enable an accurate estimate of R to be made. Also it is only possible to relate the attenuation along a path to the mean rate of rainfall along that path if there is a reasonably linear relationship between k and R. This additional requirement limits the wavelength which can be used to less than about 1·5 cm. Below about 0·8 cm the attenuation becomes too large in heavy rains. In the wavelength range which can be used for the measurements, most radar equipment is at 0·86 cm or 1·24 cm (the latter almost entirely American), and only these two wavelengths will be considered here.

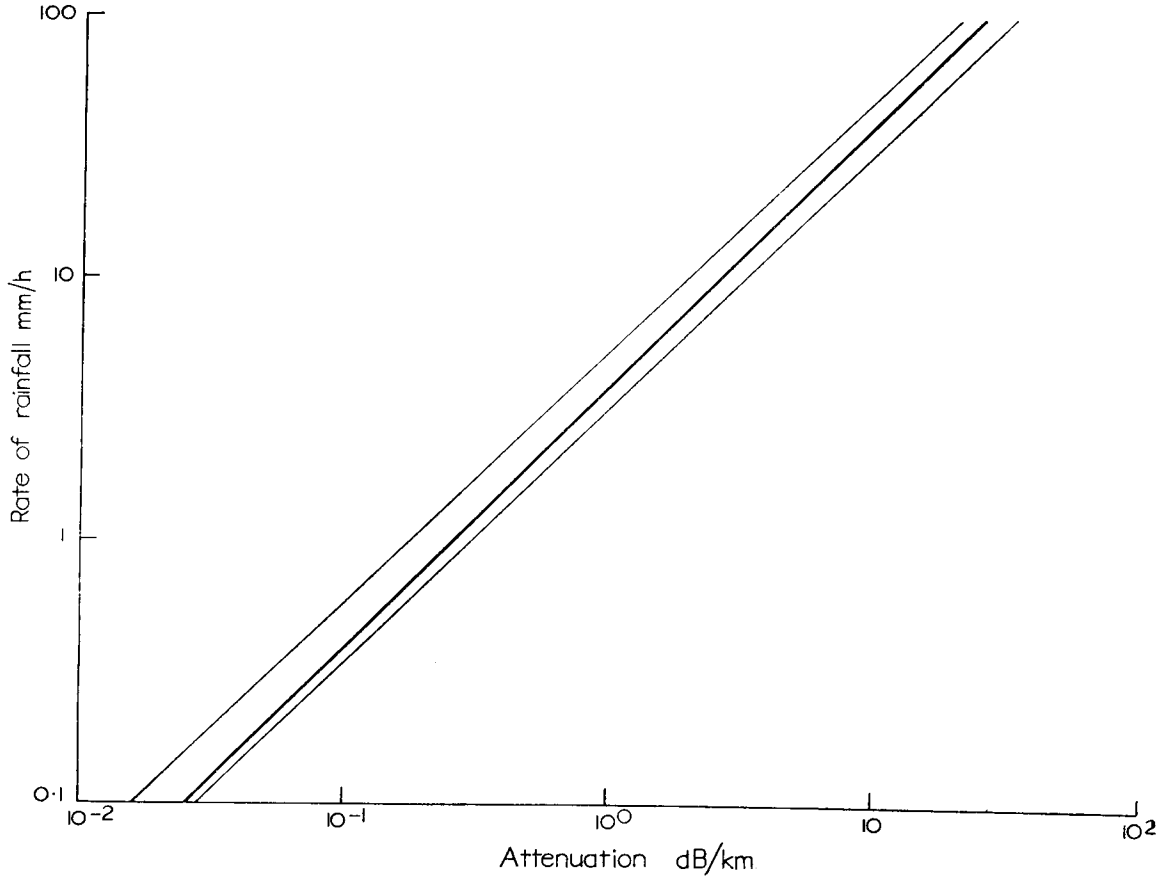


FIGURE 14. Mean k–R relationship at 0·86 cm and envelope of relationships from individual storms: London.

We have used the drop-size distributions at ground level obtained by Andrews (Part I) to calculate k at 0·86 cm using equation (8). Values of $Q(D, \lambda)$ were interpolated from Ryde and Ryde.[65] Revised values of the dielectric constant of water have been published since their paper but the differences at 0·86 cm are small (e.g. Robinson[20]), and it was not thought necessary to undertake the lengthy computations needed to recalculate $Q(D, \lambda)$. Regressions of log R on log k are given in the last column of Table II, and the overall envelope of the lines is shown in Figure 14. The analysis of the drop-size distributions for all three rain types gives

$$k = 0{\cdot}26\,R \text{ dB/km.}$$

The maximum error in a rainfall measurement in a storm, obtained by substituting known values of k into this expression, will be ± 15 per cent. The mean error will be less than 10 per cent.

The calculations have not been repeated at 1·25 cm since it would probably be necessary to recompute $Q(D, \lambda)$ at this wavelength. The scatter about a linear k–R relation will probably be slightly greater than at 0·86 cm since Haddock's best line was $k = 0{\cdot}12R^{1{\cdot}05}$. For example at $k = 12$ dB/km Haddock's line gives $R = 81$ mm/h whereas if we use $k = 0{\cdot}12R^{1{\cdot}00}$, $R = 100$ mm/h.

(*b*) *Experimental.*—Several experiments have reasonably confirmed Haddock's formulae, but because of the difficulty of knowing accurately the mean rate of rainfall along a path, it is difficult to reach definite conclusions about the scatter in the results of these experiments.

Robinson[20] working in England at 0·86 cm ensured that R was constant along his path length making measurements only when this appeared to be so on an A-scope. He measured the echo intensity in rain as a function of range and only considered results which confirmed that the rain was uniform. Below rates of 1 mm/h the attenuation was so small that the errors in the readings became comparable with the change in readings due to attenuation. Also in light rainfall it became increasingly difficult to measure the rate of rainfall. The envelope of his experimental points is shown in Figure 15. The best line through the points is $k = 0{\cdot}26R$, thus confirming the theoretical relation.

Okamura *et alii*[67] performed an experiment in Japan at 0·86 cm, over a path length of only 400 m, but since R was estimated from a single rain-gauge at one end of the path and the experiment was conducted in heavy typhoon rains, the authors admit the possibility of errors in their assumption that R was uniform over the path. The envelope of these observations is also included in Figure 15. It can be seen that the spread of both these and Robinson's experimental points is much greater than that obtained from the calculations based on Andrews' drop-size distributions and this is probably due to errors in the experimental measurements.

Another Japanese experiment has been performed by Funakawa *et alii*.[68] They worked over a range of 24 km and used four rain-gauges along the path. Consequently they were restricted to uniform rains and even then there may be small errors in their rates of rainfall along the line. The best line was $k = 0{\cdot}33R$ dB/km and their observations are included in Figure 15.

The attenuation at 1·24 cm measured by Anderson *et alii*[69] in orographic rains in Hawaii are consistently greater than Haddock's values by factors of from 1·5 to 2. This may be due to drop-size distributions which differ markedly from those of temperate latitudes. However the Z–R relationship obtained at the same time was $208R^{1{\cdot}53}$, which is very similar to those found in temperate latitudes (e.g. Figure 1). The reasons for this discrepancy

between theory and experiment have not been resolved. There do not appear to be any other experiments at this wavelength reported in the literature. In this paper Haddock's relationship will be used.

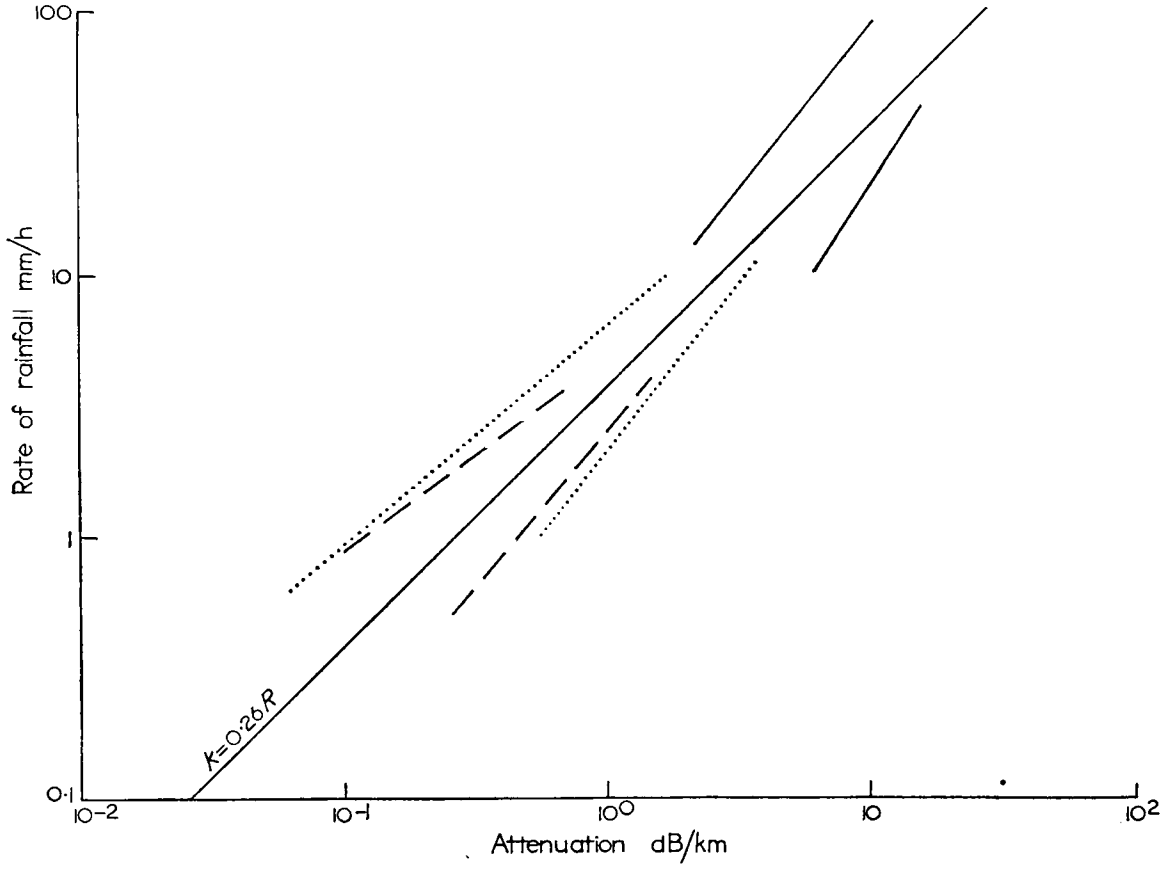


FIGURE 15. Envelopes of experimental measurements of attenuation at 0·86 cm.
—— Okamura *et alii*,[67] . . . . . Robinson,[20] — — — Funakawa *et alii*.[68]

In practice raindrops are not spherical as was assumed in the analysis of the drop-size data above. The scattering and absorption will not therefore be the same for vertically and horizontally polarized waves. Robinson[20] found that a change of polarization was not detectable in his experiment but more recent work in Japan has not supported his conclusion. Oguchi[70] found theoretically that over the range $R = 1$ to 100 mm/h, $k = 0{\cdot}26R$ for horizontal polarization and $k = 0{\cdot}22R$ for vertical polarization. A difference of this order was found experimentally by Funakawa *et alii*.[68] Consequently, although the above experiments generally confirm the relation calculated from Andrews' drop-size data, in practice account might have to be taken of the polarization used.

(*c*) *Summary.*—We have seen that at 0·86 cm theory shows $k = 0{\cdot}26R$ with only a small scatter, so that at this wavelength it is possible to measure the instantaneous mean rate of rainfall along a path by measuring the attenuation it produces. Experiments largely confirm the relationship. Haddock's results indicate that this conclusion also applies at 1·24 cm, although the probable error may be slightly greater.

3. *Practical method of measurement*

The simplest and most practical method of measuring the attenuation in rain using radar is to compare the signal strengths from a standard target at a known range in dry weather and in rain. This provides a measure of k and the mean rate of rainfall along the path between the radar and reflector can then be calculated. A rainfall amount over a period of time can be found by summing the rates of rainfall.

Measurements are only affected by the melting level when this intersects the line between radar and reflector. Attenuation in wet snow cannot be confidently related to the equivalent rate of rainfall (e.g. Robinson[20]).

4. *Other causes of attenuation*

As well as being reduced in rain a signal is also attenuated by oxygen, water vapour, fog and cloud.

Since the oxygen content of the air along a fixed path is practically constant, the attenuation ($\simeq 4\times10^{-2}$ dB/km) of the signal from a reflector does not change appreciably owing to this effect.

The attenuation in water vapour at 10°C is about $0{\cdot}01W$ dB/km (Gunn and East[19]) at 0·86 cm and $0{\cdot}023W$ dB/km at 1·24 cm. The larger value at 1·24 cm is due to an absorption band in the water vapour spectrum at 1·35 cm. Suppose there is a maximum short-term variability of 5g/m^3 of water vapour when it is raining compared with the preceding dry weather conditions. Over 20 km this results in two-way attenuations of 2 dB at 0·86 cm and 4·6 dB at 1·24 cm. Thus changes in water vapour attenuation can be important, especially at 1·24 cm and must be allowed for. In practice this could be done sufficiently accurately from a knowledge of the water vapour content (obtainable from wet- and dry-bulb readings) at the radar and assuming this value to apply over the entire range. It must be noted that the attenuation quoted by Gunn and East at 0·86 cm has not been completely verified experimentally. Funakawa *et alii*[68] measured values more than twice as large, although Tolbert *et alii*[71] were in close agreement with the Gunn and East expression.

The occurrence of fog and cloud along the path will also attenuate the signal. This could be important in hilly areas where a reflector may have to be placed in a prominent position such as a hill top in order to be visible from a radar. (Siting problems are discussed further in Section 11 of Part II.) Such sites will be enveloped in cloud at times. The attenuation in water droplet cloud is about $0{\cdot}7M$ dB/km at 0·86 cm and $0{\cdot}4M$ dB/km at 1·24 cm (Gunn and East[19]) with a typical value of M for low stratus of about 0·2 g/m^3. If cloud or fog with this liquid water content extend over 10 km of the path the contribution to two-way attenuation will be 2·8 dB at 0·86 cm and 1·6 dB at 1·24 cm. In practice then, to maintain the possible accuracy in a rainfall estimate the length of the path in cloud will have to be estimated. This should be possible to within a kilometre or two, leaving an uncertainty in the attenuation in cloud of less than $\frac{1}{2}$ dB.

5. *Reflector requirements*

The simplest type of reflector is a flat plate. However, the radar cross-section of a plate is very critically dependent on the orientation and flatness. The scattering cross-section of a square plate at a small angle α to the normal is

$$\sigma(\alpha) = \frac{4\pi l^4}{\lambda^2}\left(\frac{\sin\left(\frac{2\pi}{\lambda} l \sin\alpha\right)}{\frac{2\pi}{\lambda} l \sin\alpha}\right)^2$$

where l is a side length of the plate. This shows for example that if a plate with $l = 2$ metres is offset by only 6 minutes of arc there is a reduction of 13 dB compared with normal incidence. Such a plate would have to be accurately flat and aligned. Even then it is unlikely that it could be fixed rigidly enough to withstand movement by the wind. Collis *et alii*[72] reported considerable difficulty in an experiment using plates as reflectors.

The critical dependence on normal incidence is avoided if a sphere or tetrahedral corner reflector is used. The radar cross-section of a sphere which is large compared with the wavelength is equal to its geometrical cross-section. For a corner reflector the back-scattering cross-section is

$$\sigma_R = \frac{4\pi b^4}{3\lambda^2}$$

where b is the length of the short side and this is considerably larger than a sphere of comparable size at the wavelengths we are concerned with. It is therefore considered that a corner reflector is the most suitable reflector to use in measuring rainfall.

6. *Limitations to the rate of rainfall which can be measured*

In heavy rain the signal from a reflector may be attenuated so strongly as to be no longer detectable. This sets an upper limit to the mean rate of rainfall which can be measured.

The power received from a point target on the beam axis is

$$P_r = \frac{P_t A^2 \sigma_R}{9\lambda^2 \pi r^4}.$$

For a corner reflector

$$P_r = \frac{4P_t A^2 b^4}{27\lambda^4 r^4}.$$

Consider a radar of aerial diameter 2 metres, $P_t = 30$ kW and a minimum detectable signal of 10^{-12} watt. These are practical values at the wavelengths we are considering. The upper curves of Figure 16 show the highest mean rate of rainfall along different path lengths which can be measured using such a radar with a reflector of short-side length either 50 cm or 200 cm. The latter is considered to be about the largest practical size. The attenuation produced by 10 g/m^3 of water vapour is included in the diagram, this value being typical of summer in this country.

The figure indicates that heavy rains can be accurately measured over longer path lengths with 1·24-cm radar. However, even at this wavelength measurements can be made only over a limited range. There are apparently no statistics available of the rates of rainfall which can occur along a line in this country, and so no definite statement can be made of the probability that all rains could be measured over a stated path length with a particular reflector. One would, however, expect a mean rate of 10 mm/h over 30 km (equivalent for example to 100 mm/h over 3 km) to be rare in this country, so all rains could probably be measured over about 30 km using a 200-cm reflector at 1·24 cm. At 0·86 cm the signal from a 200-cm reflector would probably be attenuated to below the minimum detectable level in heavy rains over a path length of more than 10 km.

If a signal is attenuated to below the minimum detectable level then this shows the rainfall to be greater than a known rate. Since these heavy rates would be generally short lived over the path between radar and reflector and may not be far above the upper limit which can be measured, useful rainfall measurements can be made over a greater range

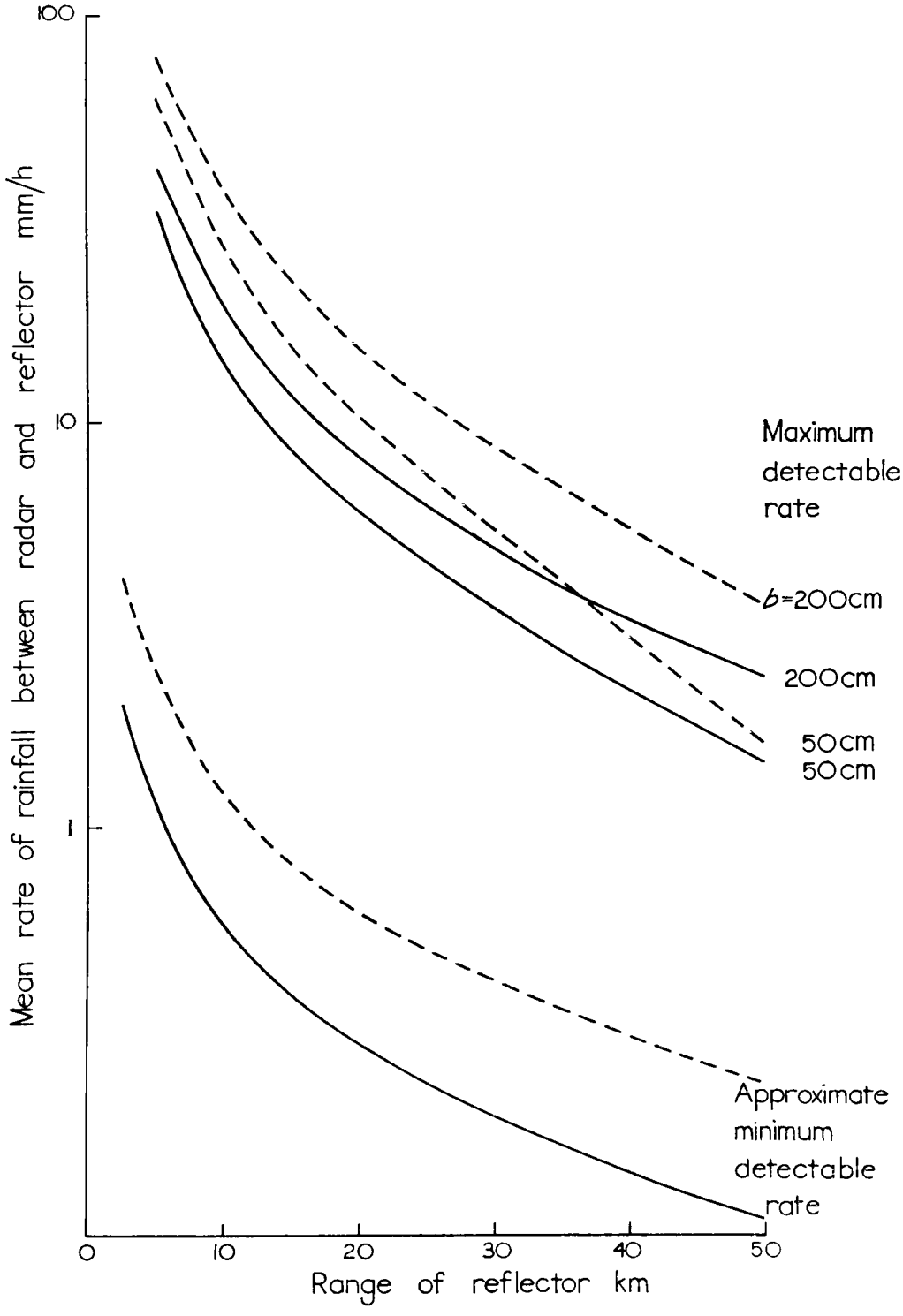


FIGURE 16. Limits to the rates of rainfall measurable using a corner reflector.

—— 0·86 cm, - - - - 1·24 cm. Radar parameters: P_t = 30 kW, P_{MIN} = 10^{-12}W, A = π. Water vapour content of air 10 g/m^3. b = length of short side of reflector.

than suggested by Figure 16 and the preceding paragraph. For example, if as much as a 50 per cent error in higher rainfalls can be accepted, then a 0·86-cm radar could probably measure rainfall out to 30 km using a 200-cm reflector. As an illustration, suppose 9 mm/h over 30 km lasts for 15 minutes in a storm. One could infer that this was a mean rainfall in excess of 5 mm/h (the rate for which the signal is no longer detectable), and on the basis of this minimum figure one would underestimate the amount of rain in the 15 minutes by 44 per cent or 1 mm. For the entire storm the percentage error would be less since some of the rain would be accurately measurable. At 1.24 cm measurements could probably be extended to 50 km if the same error can be accepted. The error would always be an underestimate.

However, it must be emphasized that even if accurate knowledge of the larger rates of rainfall is not required, the range over which rainfall can be measured by the attenuation method is limited, even in this country. It would be invalidated in tropical rains, without quite spectacular improvements in P_t, P_r, A or b.

It is considered that a change of 3 dB in the signal from the reflector is about the minimum that can be confidently be attributed to rain. The lower curves of Figure 16 show the rate of rainfall needed to produce this attenuation.

7. *Receiver requirements*

The radar must be quantitative for all signal intensities received from a reflector. As an example of the range of signals to be handled, the echo from a 200-cm reflector at 20 km, using the 0·86-cm radar assumed in the previous section, will vary over 80 dB between dry weather and heavy rain.

The characteristic of the pre-amplifier of a radar is linear over only about 45 dB of input power, and saturation generally occurs in the region of 70–80 dB. Because a dynamic range of 80 dB is necessary the balance of 35 dB or even more of the attenuation would need to be inserted at the radio frequency (RF) stage. The attenuator(s) could probably be made self-recording, that is a continuous record of the power received at the aerial from the reflector could be obtained and this, as we have seen, is directly relatable to the mean rate of rainfall along the path.

An alternative system would be a combination of RF and intermediate frequency (IF) attenuators with sufficient RF attenuation to restrict the larger signals to the linear portion of the pre-amplifier characteristic. If the amount of RF attenuation is known, then quantitative measurement can be made in the IF part of the receiver.

8. *Effect of permanent echoes, anomalous propagation and rain echoes*

Since the reflector will be close to the ground, echo will also be received from the ground within the pulse volume. It is expected that the echo from the reflector will be the greater and steadier, and will be easily distinguishable. In practice, a small ground return will probably contribute to the reflector echo, but preliminary measurements made from Malvern of a reflector on the side of a hill suggest that this additional echo is sufficiently constant for it not to interfere with the steady reflector signal. At times, especially in association with an inversion below about 1000 feet (e.g. Funakawa *et alii*[68]) anomalous propagation occurs, which is apparent as a fading of the reflector signal. In such conditions it will be very difficult to obtain a meaningful measure of the signal strength. However, during rain an inversion below about 1000 feet is unusual, except occasionally near thunderstorms. Consequently the occurrences of fading during rain should be rare.

When it is raining at the target, the radar will receive a signal from the target and also from the precipitation within the pulse volume. Consider the relative returns from the two sources in a particular case of $\lambda = 0{\cdot}86$ cm, $h = 0{\cdot}2\,\mu$s $= 6 \times 10^3$ cm, aerial diameter 2 m and range 20 km. Assume a large Z value of 10^6 mm^6/m^3. Then the signal from the target is

$$P_{\mathrm{TAR}} = \frac{P_t A^2 \sigma}{9\pi\lambda^2 r^4} = y\sigma_R, \text{ where } y = \frac{P_t A^2}{9\pi\lambda^2 r^4}$$

and for a corner reflector of $b = 200$ m

$$\sigma_R = 9{\cdot}1 \times 10^7 \text{ m}^2$$

so

$$P_{\mathrm{TAR}} = 9{\cdot}1 \times 10^7\, y.$$

Assuming the rain completely fills the pulse volume, then it can be shown, using the radar equation (3),

$$\frac{P_{\mathrm{RAIN}}}{P_{\mathrm{TAR}}} \simeq 10^{-4}.$$

Consequently the signal from the rain will be about 40 dB below that received from the target and can be neglected. This conclusion applies to all practical rates of rainfall and ranges likely to be encountered in this country.

9. *Radar stability*

Radar stability has already been mentioned in Part IA. If the signal from a reflector in dry weather were measured several times a week, it would serve as a check on the general radar performance. However, because of stability variations there may be an error of 1 or 2 dB in any reading in rain. The effect of such variations is eliminated if two reflectors are used at different ranges along the same path, the variations in the difference between the signals from the two reflectors serving as a measure of rainfall. Or the radar can be frequently calibrated.

Over a range of 20 km an uncertainty of ± 2 dB corresponds to an error of $\pm 0{\cdot}05$ dB/km in k. This is unimportant at higher rates of rainfall and even at 1 mm/h amounts only to ± 20 per cent. Hence, for ranges of more than 20 km the error arising from the instability of the radar should be insufficient to warrant the use of two reflectors. For shorter ranges, especially in light rainfalls, the instability is an important source of error.

10. *Experimental investigations*

Although much work has been done in Japan on the measurements of signals from reflectors at 8 mm, the possibilities of measuring rainfall in this manner do not appear to have been discussed. Rather, the work has been concerned with investigating the possibility of communication links in this waveband.

The only other published report of which the author is aware is a recent one by Collis *et alii*[72]. Between 1960 and 1963 they carried out an experimental programme to study the possibilities of measuring rainfall by reference to the attenuation at 0·86 cm. Their main conclusion is that "While the relationship between k and R shows a marked and systematic relationship generally corresponding to the theoretical values, this relationship is not sufficiently consistent to warrant use in the measurement of rainfall". This result is not one which would be expected from the drop-size analysis summarized in Table II and Figure 14.

The experimental set-up they used is shown in Figure 17. The rain-gauge network was sufficiently dense to give a reliable estimate of rainfall along the line and since measurements were only made between the two reflectors A and B, variations of radar stability were not important (see Section 9). However, plane reflectors were used (except at B toward the end of the project), and it has already been shown that the echo cross-section of such a reflector is critically dependent on orientation. The report states that at times the wind prevented measurements being made. It seems at least possible that some of the variation in the k–R relations may have been due to changes in the echo cross-section and especially in the reflection at reflector A. Drop-size distributions in California, the location of the experiment, may also be more variable than in London. In the author's opinion the conclusions from this experiment cannot be applied *a priori* to this country.

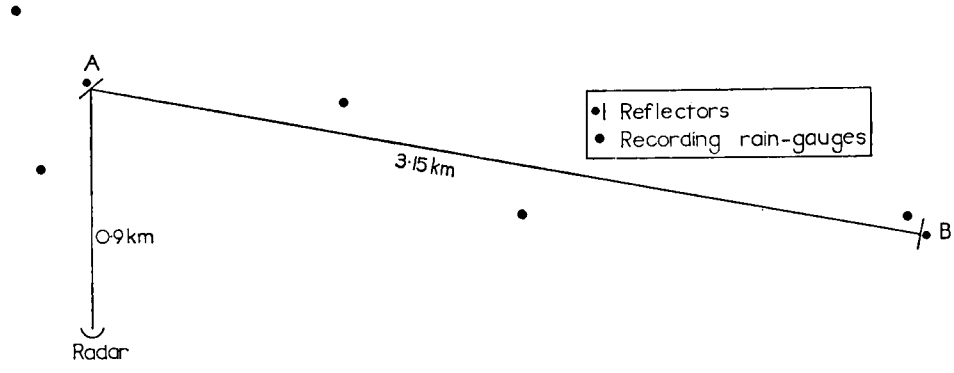


FIGURE 17. Experimental network used by Collis *et alii.*[72]

11. *Measurement of rainfall over an area using reflectors*

It has been shown that it should be possible to measure the mean rate of rainfall along a line. Measurements along several lines over a sector would provide an approximate estimate of the mean rate of rainfall over that sector. The accuracy of the estimate depends upon the variability of the rain and the number of lines within the area—that is, the number of reflectors used. It is believed that a satisfactory compromise giving a useful coverage with a modest number of reflectors, would be provided by a reflector spacing of 5 to 10 km along the circumference of the section. It is realized that with this spacing some slow-moving convective cells may be missed entirely. In practice, the topography of an area will prevent any regular spacing of the reflectors.

The method of estimating rainfall over an area from a knowledge of the mean rates of rainfall along known path lengths will now be considered in more detail. As an example, suppose a radar at O views two reflectors A and B as shown in Figure 18. Let the mean rates of rainfall be R_1 along l_1 and R_2 along l_2. It is required to find the best estimate of rainfall over the area AOB.

Over the area AOB the mean rate of rainfall R_A is

$$R_A = \frac{\int_0^{\phi} l^2 \, R \mathrm{d}\theta}{l_1 l_2 \sin \phi} \tag{10}$$

where

$$l = l_1 \frac{\sin \gamma}{\sin (\gamma + \theta)} \tag{11}$$

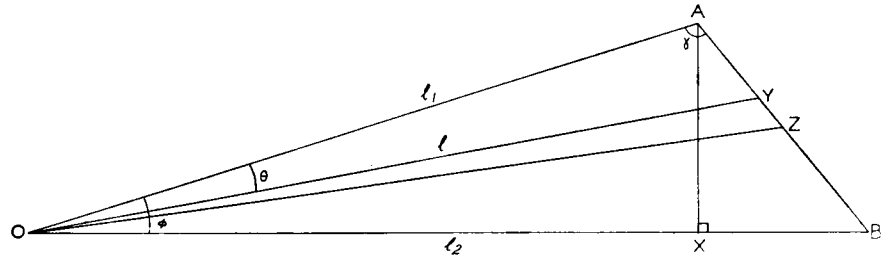


FIGURE 18. Estimation of rainfall over an area.

To solve the integral an assumption must be made concerning the variation of R with θ. Several assumptions can be made and a few will be considered briefly.

(i) The simplest assumption is that the mean rate of rainfall is $(R_1 + R_2)/2$ over the area. The disadvantage of this is that it gives equal weight over lines of different lengths.

(ii) Another assumption is to assume the mean rate of rainfall is $(l_1 R_1 + l_2 R_2)/2\bar{l}$ where $\bar{l}$ is the mean l for the area. As an example of the error which can occur with this assumption consider the case when the rainfall is uniform over the area and so $R_1 = R_2 = R_A \neq R_A(l_1 + l_2)/2\bar{l}$ for the area of the diagram.

(iii) Suppose R varies linearly with θ. This simple assumption is a little more realistic than the previous two and will be used here.
Let

$$R = R_1 + \alpha \theta \tag{12}$$

where

$$\alpha = \frac{R_2 - R_1}{\phi} .$$

By inserting (11) and (12) into (10) it can be shown that

$$R_A = a R_1 + b R_2 \tag{13}$$

where

$$a = \frac{l_1}{l_2} \frac{\sin^2 \gamma}{\sin \phi} \left(\cot \gamma - \frac{1}{\phi} \ln \frac{l_1}{l_2} \right)$$

and

$$b = \frac{l_1}{l_2} \frac{\sin^2 \gamma}{\sin \phi} \left(\frac{1}{\phi} \ln \frac{l_1}{l_2} - \cot (\gamma + \phi) \right)$$

so that a and b are functions only of the geometry of the area. An expression similar to (13) can be found for other shaped areas bounded by l_1 and l_2 provided the l can be expressed in terms of θ, but it is likely that the integral would have to be solved graphically.

The method can be extended to an area covered by several reflectors by finding R_A for each segment as above. If the mean rates of rainfall are R_{A_1}, R_{A_2} etc. over areas A_1, A_2 etc., then

$$R_A = \frac{R_{A_1} A_1 + R_{A_2} A_2 + R_{A_3} A_3 \ldots}{A_1 + A_2 + A_3 \ldots} \tag{14}$$

Errors in the estimate of rain over an area may arise from two causes in addition to those already considered: (i) that the assumption of the variation of R with θ is not correct: (ii) that details of the variation of rainfall along the line are not known.

We will consider an extreme example of the error which might occur. In Figure 18 suppose $l_1 = 20$ km, $l_2 = 30$ km and AX = 6 km. Then from (13)

$$R_A = 0{\cdot}444\, R_1 + 0{\cdot}556\, R_2$$

Suppose a convective cell with $R = 100$ mm/h over an area of radius $\frac{1}{2}$ km is centred over the radar, and let it be dry over the rest of the area. Then

$$R_1 = 2{\cdot}5 \text{ mm/h}, \qquad R_2 = 1{\cdot}66 \text{ mm/h}$$

and

$$R_A = 2{\cdot}03 \text{ mm/h}$$

whereas the mean rate over the area is in fact only 0·05 mm/h. The areal estimate is in considerable error but it must be emphasized that this is an extreme case. At the other extreme the cell could go undetected if it were positioned between the reflector azimuths, when the actual mean rate over the area would be 0·87 mm/h.

Errors are unlikely to be of this magnitude in widespread rain and in cases when convective cells move through the area—errors in this latter case tending to cancel out.

The accuracy of this method of measuring rainfall should be compared with the density of a rain-gauge network needed to achieve the same accuracy, but no data on this point have been published.

The extension of line measurements to a measurement over an area would often be limited by the topography. It is difficult to imagine many catchments over which a sufficient number of reflectors could be suitably placed to enable a confident areal measurement to be made.

12. *Practical application of measuring over an area*

The instrumental requirements are broadly similar to those stated in Section 17, Part IB, para. 1—viz. The radar is required to process the incoming voltages automatically and register accrued rainfall over an area.

Let P_{DRY} = received power in no rain

P_{RAIN} = received power in rain.

The attenuation of the signal from a reflector is

$$10 \log \frac{P_{\text{DRY}}}{P_{\text{RAIN}}} \text{ dB}$$

and therefore

$$k = \frac{10}{2l} \log \frac{P_{\text{DRY}}}{P_{\text{RAIN}}} \text{ dB/km.}$$

If V is the voltage at the aerial ($P \propto V^2$), then

$$k = \frac{10}{l} \log \frac{V_{\text{DRY}}}{V_{\text{RAIN}}},$$

or assuming $k = 0{\cdot}26R$

$$R = \frac{10}{0{\cdot}26l} \log \frac{V_{\text{DRY}}}{V_{\text{RAIN}}}. \tag{15}$$

In a measurement over an area it is envisaged that the radar automatically 'reads' the signal from each reflector in turn. From equations (11) and (12)

$$R_A = \alpha_1 R_1 + \alpha_2 R_2 + \ldots \tag{16}$$

where α_1 and α_2 etc. are known constants. The instrumentation is required to compute R_A automatically from measurements of the signal strength from several reflectors. By summing R_A over a period of time an estimate of rainfall amount can be made. Circuitry is therefore required which will solve equation (15) for each reflector and perform the additions of equation (16). In equation (15) it should be noted that although l and E_{DRY} will be known constants for a given reflector, they will vary from reflector to reflector.

It is not the intention of this paper to describe in detail the components required to perform the computation. The block diagram (Figure 19) indicates one means by which accrued rainfall could be measured. The diagram indicates how a voltage proportional to R for each reflector can be generated. In turning from one reflector to another, the gain control and attenuator must be automatically changed to their appropriate values.

An alternative system may be to use RF attenuators. It would perhaps be possible to measure the attenuation required to reduce E_{RAIN} to a fixed level and compare this with the (known) attenuation required to reduce E_{DRY} to the same level. The output of this system could presumably be a voltage proportional to the attenuation in dB caused by the rain. This replaces the block diagram up to and including the gain control. The remainder of the system would be retained to produce an output proportional to the rainfall over the area.

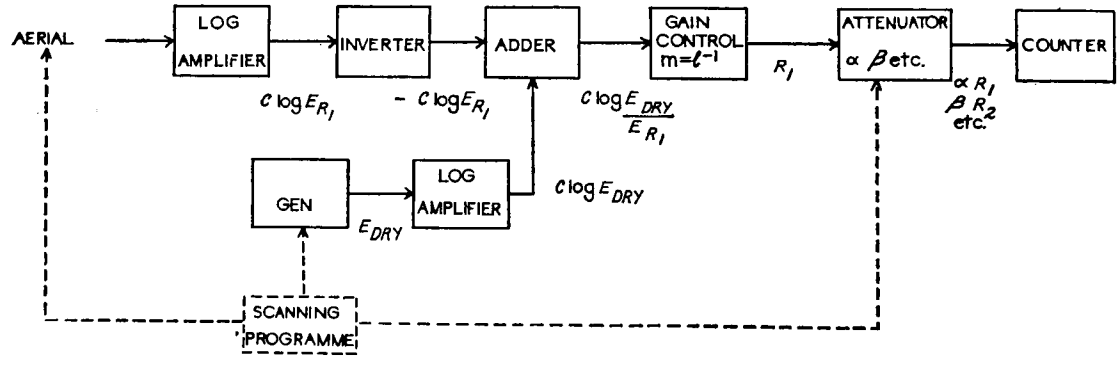


FIGURE 19. Block diagram of one type of computer for measuring rainfall over an area.

13. *Possibilities of measuring rain using one-way attenuation*

This system would consist of a transmitter at one end and a receiver at the other end of the path. No reflector is needed. The path length of the radiation is equal to the range and not twice the range as above, so that the attenuation is effectively halved for a given

range. Continuous wave radiation, rather than pulsed radiation, would be used with resulting simplification of the equipment. The major drawback of the method is that an electrical supply is required at both ends of the path. It is considered that a battery supply could be used if necessary but the working life between charges would be about 30 hours, and in remote areas especially, this would be impracticable. The simplicity of a reflector compared with a separate transmitter favours the reflector method.

14. *Conclusions and recommendations*

It has been shown that an accurate estimate of the mean rate of rainfall along a line can probably be obtained by comparing the echo from a corner reflector in wet and dry weather, using a wavelength between 0·8 and 1·5 cm. At these wavelengths the attenuation is directly proportional to the mean rate of rainfall along the path, i.e. $k \propto R$. However, Collis *et alii* found in California a larger day-to-day variation than would be expected from an analysis of drop-size data obtained in London. It has been suggested that their results might not hold in Britain.

There is a limit to the range over which heavy rainfall can be measured (Figure 15), since the reflector signal may be attenuated to below the minimum detectable level. This range is greater than that over which rainfall can be accurately measured from the echo intensity when the melting layer is low. Nevertheless, this limitation in range is a restriction of the method. However, when the signal is no longer detectable the mean rate of rainfall is known to be above a certain value, and this may be sufficient indication of the likelihood of flooding.

In a point measurement of rainfall using a rain-gauge, the measurement is assumed to apply over an area, the confidence in the estimate being a function of, among other factors, the size of the area. Analogous to this, the measurement over a line can also be assumed to hold over an area about that line, and this area is most likely greater than when using a rain gauge. In other words, a reflector provides a better areal estimate than a single rain-gauge.

It has also been shown how measurements can be extended to a larger area, but doubts have been expressed as to the practicability of this over many catchments since it requires a series of reflectors all visible from the radar. Circuitry which would automatically compute and register accrued rainfall has been suggested.

In that it is not a true area integration, the method is considered to be inferior to the echo intensity method discussed in Part I, but at times it can be considerably more accurate. This is especially true over short ranges, when all rates of rainfall can be measured, whereas no rain can be measured within about the first 15 km in the echo intensity method. The attenuation method is also more accurate over larger ranges in moderate rains when the melting layer is low. However, in an automatic measurement of areal rainfall the circuitry is at least as complex as in Part I, and accuracy is also limited. In general the increased complexity involved in measuring signals from several fixed reflectors would make this method less suitable for an areal rainfall measurement than the more versatile echo intensity method. The method is most suitable for measuring rainfall along a line and assuming this to hold over a restricted area. The line could be one liable to flooding and in particular could be along a valley.

In the author's opinion Collis's experiment cannot *a priori* be assumed to indicate the accuracy of this method of measuring rainfall, at least in this country, and an experiment has been planned at the Meteorological Research Unit, Malvern, to investigate this accuracy.

Two corner reflectors have been positioned along the same line of sight, 7 km apart, and it is intended to measure the attenuation at 0·86 cm between them. This will be compared with the mean rate of rainfall estimated from six rain-gauges along the line. The gauges are part of the Winchcombe network and it is hoped from this to determine the confidence with which a measurement over a line can be applied to an area about that line.

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